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Modified strip saturation model for a cracked piezoelectric strip

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ABSTRACT

Purpose: The investigations aim to propose a model for arresting an electrical opening of a crack which weakens a narrow, poled and infinite piezoelectric strip. The edges of the strip are subjected to uniform, constant anti-plane stresses and in-plane electrical displacements.

Design/methodology/approach: The loads applied at the edges of the strip open the crack in a self-similar fashion. Consequently at each tip of the crack a saturation zone protrudes. To stop the crack from further opening the rims of developed saturation zones are subjected to normal, cohesive linearly varying saturation limit electric displacement. The edges of the strip are subjected to anti-plane deformation and in-plane electrical displacement. Fourier integral transform method employed reduces the problem to the solution of a Fredholm integral equation of second kind.

Findings: The electrical displacement, stress intensity factor, the saturation zone length, crack opening displacement and crack growth rate have been calculated. The results obtained presented graphically, analysed and concluded.

Research limitations/implications: The ceramic used for strip is being assumed to be electrically more brittle. The investigations are carried at this level in the present paper. Also the small scale electrical yielding is considered. Consequently the developed saturation zone is proposed to lie in a line segment ahead of crack.

Practical implications: Piezoelectric ceramics being widely used as transducers. Their wide utility has prompted to study many attires of such ceramic and one such attire is fracture mechanics of these ceramics.

Originality/value: The paper gives an assessment of the electrical load necessary to arrest the electrical crack opening. The investigations are useful to smart material design technology where sensors and actuators are manufactured.

Keywords: Applied mechanics; Crack growth rate; Piezoelectric ceramics; Strip saturation model

METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

Following the theory of linear piezoelectricity, the problem of determining the singular stress and electric fields in an orthotropic piezoelectric ceramic strip containing a Griffith crack under longitudinal shear is considered in [1]. The anti-plane dynamic fracture problem of a cracked infinite piezoelectric strip is studied [2] using integral transform mehod. Transient dynamic loads are considered in the paper. The electric-field induced stress intensity factor in a piezoelectric medium of limited electrical polarization is evaluated [3] based on a strip-saturation model of the Dugdale-type. Here particular emphasis is placed on the effect of the saturation condition on the near tip field and the stress intensity factor. Using elasto-plastic fracture mechanics approach for crack growth simulation in the presence of residual stress fields is presented [4] using a boundary element method. A numerical analysis is conducted [5] to establish basic toughening mechanism and fracture behaviour of an interlayer–toughened composite material, containing particular modified toughened layers. The aim of this analysis is to examine the influence of the particles on the plastic zone size that develops in front of crack tip. A piezoelectric strip with permeable edge cracks normal to the strip boundaries is analysed [6] under uniform anti-plane mechanical shear and in-plane electric loading. The electrical non-linear behaviour of an anti-plane shear crack in a functionally graded piezoelectric strip is studied [7] by using strip saturation model with in the frame work of linear electro-elasticity. A mode III crack problem for a functionally graded piezoelectric material strip is dealt in [8]. The electric field saturation model is applied [9] to the fracture prediction of a piezoelectric material containing electrically impermeable cracks. A crack arrest model [10] is proposed for a poled piezoelectric plate weakened by a finite hairline straight crack. The coating of a piezoelectric material with sub-micron size diamond particles has enabled to produce truly flat substrates and machine micro-features into electronic and optical materials using a new manufacturing process known as "piezoelectric micro-grinding". The paper [11] describes the principle of the process of micro-grinding using coated piezoelectric materials. A modified strip yield model is proposed [12] for a cracked piezoelectric strip subjected to in-plane mechanical and electrical loadings when developed slide zones are arrested by quadratically varying yield point stress.

The Acoustic Emission (AE) method is employed to study [13] the processes of surface crack initiation and evaluation in surface protective coatings. A finite element formulation [14] is developed for modelling the dynamic and static response of laminated plates containing discrete piezoelectric ceramics subjected to both mechanical and electrical loadings. An anti-plane shear problem [15] for a cracked functionally graded piezoelectric material layer bonded to two piezoelectric half-planes is analyzed using integral transforms.

2. Fundamental formulation

As is well-known for an out-of-plane displacement components, u_z, and in-plane electric field components E_i, $(i = x, y, z)$ may be defined as

$$u_x = 0, u_y = 0 \text{ and } u_z = w(x, y) \tag{1}$$

$$E_x = E_x(x, y) = -\phi_{,x}, E_y = E_y(x, y) = -\phi_{,y} \text{and } E_z = 0 \tag{2}$$

where ϕ denotes the electrical potential and comma after the function denotes the partial differentiation with respect to argument following it.

The constitutive equations are

$$\sigma_{xz} = c_{44} w_{,x} + e_{15}\phi_{,x}; \; \sigma_{yz} = c_{44} w_{,y} + e_{15}\phi_{,y} \tag{3}$$

$$D_x = e_{15} w_{,x} - \varepsilon_{11}\phi_{,x}; \; D_y = e_{15} w_{,y} - \varepsilon_{11}\phi_{,y} \tag{4}$$

$$\gamma_{xz} = w_{,x}; \; \gamma_{yz} = w_{,y} \tag{5}$$

Governing equation for this case reduce to

$$\nabla^2 w = 0, \; \nabla^2 \phi = 0 \tag{6}$$

where ∇^2 is the Laplacian operator in two-dimensions.

Using Fourier cosine transformation the solution of equation (6) in general, may be written as

$$w(x, y) = \frac{2}{\pi}\int_0^\infty [A_1(\alpha)\cosh(\alpha y) + A_2(\alpha)\sinh(\alpha y)]\cos(\alpha x)\, d\alpha + a_0 y, \tag{7}$$

$$\phi(x, y) = \frac{2}{\pi}\int_0^\infty [B_1(\alpha)\cosh(\alpha y) + B_2(\alpha)\sinh(\alpha y)]\cos(\alpha x)\, d\alpha - b_0 y, \tag{8}$$

where a_0 and b_0 are arbitrary constants determined by edge boundary conditions and $A_i(\alpha)$ and $B_i(\alpha)\,(i = 1, 2)$ are the unknown functions determine from other boundary conditions.

The constitutive equations for vaccum reduce to

$$D_x^V = \varepsilon_0 E_x^V, \; D_y^V = \varepsilon_0 E_y^V \tag{9}$$

and governing equation reduce to

$$\nabla^2 \phi^V = 0 \tag{10}$$

where superscript V denotes that the quantities refer to vaccum and ε_0 denotes the electrical permitivity of vaccum. The solution of equation (10) may be written as

$$\phi^V(x, y) = \frac{2}{\pi}\int_0^\infty C(\alpha)\sinh(\alpha y)\cos(\alpha x)\, d\alpha, \; 0 \le x < c \tag{11}$$

3. Statement of the problem and solution

The schematic configration of the problem is shown in Figure 1 above. An infinitely long piezoelectric ceramic exhibiting symmetry of a hexagonal crystal of class 6 mm with respect to principal axes x, y and z, occupies the region $-\infty < x < \infty, -h \le y \le h$ and is thick enough in z-direction to allow the anti-plane state. The strip is cut along a hairline straight crack occupying the region $-c \le x \le c, \; y = 0$. Rims of the crack open in self-similar fashion due to the combined in-plane electrical loading anti-plane deformation. Consequently a saturation zone protrudes ahead of each tip of the crack. To stop the crack from further opening the rims of the developed zones are subjected to in-plane linearly varying saturation limit electric displacement $(x/c)D_s$. Mathematically these conditions may be written as

(i) $D_y(x, 0) = \frac{x}{c} D_s\, H(x - c)$ for $0 \le x < a$

(ii) $w(x, 0) = 0$ for $a \le x < \infty$

(iii) $\phi(x, 0) = 0$ for $c \le x < \infty$

(iv) $E_x(x, 0) = E_x^V(x, 0)$ for $0 \le x < c$

(v) $\gamma_{yz}(x, h) = \gamma_h, \; D_y(x, h) = D_h$

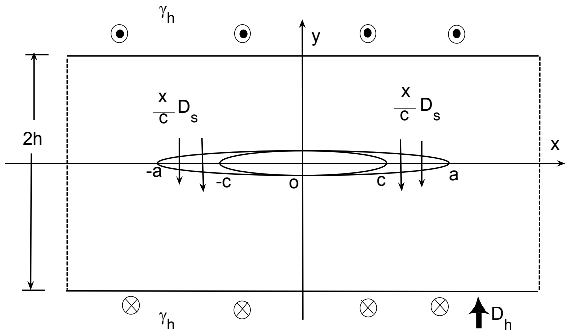

Fig. 1. Schematic presentation of the problem

Desired potential are written using equation (7,8). The constants a_0 and b_0 are obtained, using boundary conditions (v) and equations (3,4) as

$$a_0=\gamma_h \;,\; b_0=(D_h-e_{15}\gamma_h)/h \tag{12}$$

The unknown function $A_i(\alpha)$ and $B_i(\alpha)$, (i = 1,2) are obtained using boundary condition (i to iv) together with equations (3 to 11) and introducing two new parameters $\psi_1(\xi)$ and $\psi_2(\xi)$ as

$$\mathrm{A}_1(\alpha)=\frac{\pi a^2}{2}\int_0^1\sqrt{\xi}\;\Psi_1(\xi)J_0(a\alpha\xi)\,d\xi \tag{13}$$

$$\mathrm{B}_1(\alpha)=\frac{\pi c^2}{2}\int_0^1\sqrt{\xi}\;\Psi_2(\xi)J_0(c\alpha\xi)\,d\xi \tag{14}$$

where $J_0(\;)$ is the Bessel function of first kind.

These yield $B_1(\alpha)=0,\; B_2(\alpha)=0;$

$$A_2(\alpha)=-\tanh(\alpha h)\,A_1(\alpha) \tag{15}$$

and $\psi_1(\xi)$ is determined from following Fredholm integral equation of second kind

$$\Psi_1(\xi)\;+\;\int_0^1 K(\xi,\eta)\Psi_1(\eta)\;d\eta\;=\;\begin{cases}\sqrt{\xi}\,d_0/e_{15}\,, & \xi<\dfrac{c}{a}\\[2ex](d_0\sqrt{\xi}-2D_s a\sqrt{\xi}\;\sqrt{\xi^2-c^2/a^2}\;/\pi)/e_{15}, & \dfrac{c}{a}<\xi<1\end{cases} \tag{16}$$

where

$$K(\xi,\eta)=\sqrt{\xi\eta}\int_0^\infty\alpha\{\tanh(\alpha h/a)-1\}J_0(\alpha\xi)J_0(\alpha\eta)\,d\alpha \tag{17}$$

$$d_0=e_{15}a_0+\varepsilon_{11}b_0 \tag{18}$$

4. Applications

(a) The saturation zone length is determined using the condition that the electric displacement stress intensity remain finite at every point of the body which yields

$$\left(\frac{a}{c}\right)^2=\left(\frac{\pi}{2}\,\frac{d_0-e_{15}\,T(h/a)}{D_s}\right)^2+1 \tag{19}$$

(b) Crack opening displacement at any point x of the crack is obtained from

$$COD(x)=\frac{8D_s}{\pi e_{15}}\left[\sqrt{a^2-x^2}\sqrt{\frac{a^2}{\mathrm{c}^2}-1}\;-\frac{1}{c}\int_x^a\frac{t\sqrt{t^2-c^2}}{\sqrt{t^2-x^2}}\,dt\right] \tag{20}$$

(c) Crack growth rate is obtained under the assumption that cyclic load conditions may be obtained from static condition by substitution

$$D_h\to\frac{\Delta D_h}{2},\,D_0\to\frac{\Delta D_0}{2},\,D_s\to D_{sc},\,D_T=D_C=\left|\Delta COD\right| \tag{21}$$

where Δ denotes the cyclic loading conditions, D_0 denotes the electrical displacement at zero mechanical loading, D_{sc} denotes the saturation limit of cyclic loading of electric displacement, D_T denotes the accumulated saturation zone displacement and D_c is its critical value. This may finally be written as

$$\sum\left|\Delta COD\right|=\frac{2}{\delta c}\int_c^a\Delta COD(x)\,dx \tag{22}$$

The crack growth rate, $dc/dN=\delta c$ is obtained using

$$dc/dN=\{\pi\,(\Delta K_D)^4\}/\{192\gamma e_{15}D_{sc}^2\}, \tag{23}$$

where $\Delta K_D=\{\,\Delta D_h-2e_{15}\,T\,(h/a)\}\sqrt{\pi c}$,

$\mathrm{T(h/a)}=\int_0^1\mathrm{K}(1,\eta)\psi_1(\eta)\,d\eta,\;\gamma=D_c\,D_s$

5. Case study

The model proposed above is applied to study the crack arrest model for the piezoelectric strip of ceramics $BaTiO_3$, PZT-4 and PZT- 5H.

Figure 2 depicts the variation of crack growth rate vs. strip width to crack length. It is observed the crack growth rate drops drastically and then stabilizes as strip width is increased for a fixed crack length.

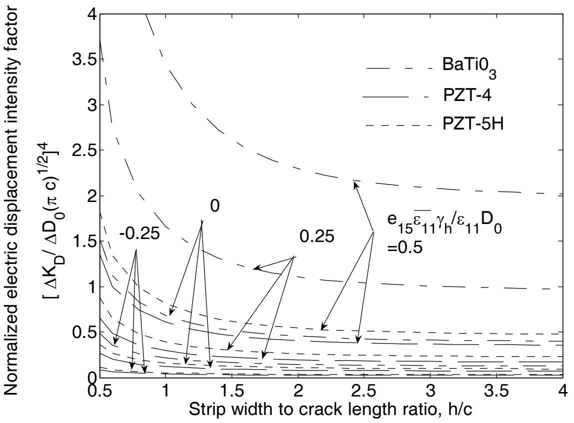

Fig. 2. Plots crack growth rate vs. h/c

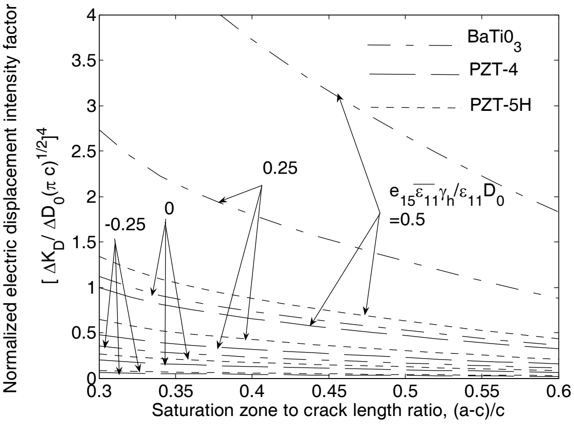

Fig. 3. Plots crack growth rate vs. (a-c)/c variation

Variation of crack growth rate vs saturation zone to crack length ratio is plotted in Figure 3. It is observed that as this ratio increases the crack growth rate reduces. In both the figures it is observed that crack arrest is material dependent.

6. Conclusions

A crack opening arrest model is proposed for a cracked piezoelectric strip.The case study on 6 mm crystal exhibiting the symmetry about principal axis shows that it is possible to reduce the crack growth rate by the proposed model.

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