



# Dynamic rigidity and loss factor prediction for composite layered panel

**B. Diveyev\*, A. Smolskyy, M. Sukhorolskyy**

Lviv National Polytechnic University, Ukraine

\* Corresponding author: E-mail address: divbogl@lviv@yahoo.com

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## ABSTRACT

**Purpose:** The study aims to predict the dynamic rigidity and loss factor properties of composite laminated plates.

**Design/methodology/approach:** Dynamic rigidity and loss factor properties have been determined by using a numerical schemes based on multi-level theoretical approach.

**Findings:** The present paper is the first attempt at proposing a novel adaptive procedure to derive dynamic rigidity and damping parameters for sandwich plate's vibration.

**Research limitations/implications:** In the future the extension of the present approach to sandwich plates with different core materials will be performed in order to test various sandwich design.

**Practical implications:** Structures composed of laminated materials are among the most important structures used in modern engineering and especially in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications.

**Originality/value:** The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical or approximation method

**Keywords:** Computational material science; Composite materials; Laminated plates; Dynamic rigidity; Damping; Loss factor

## METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

### 1. Introduction

The passive device was designed to use three technologies packaged into one device to provide increased transmission loss of a panel covering a frequency range of 10–1000Hz. Each technology is known to work for a specific frequency range: piezoelectric active control actuators for low frequencies (<200)Hz, distributed vibration absorbers for medium frequencies (75–250Hz), and constrained layer damping for high frequencies (>200Hz). By combining these technologies and packaging them into a single device, control over an extended bandwidth can be achieved. The experimental design of last two technologies was discussed in [1]. The constrained damping layer method is now under discussion.

Since the late 1950's many papers have been published on the vibration of sandwich structures [2-9]. All the models discussed

so far are based on the following assumptions: (a) the viscoelastic layer undergoes only shear deformation and hence the extensional energy of the core is neglected; (b) the face sheets are elastic and isotropic and their contribution to the shear energy is neglected; and (c) in the face-sheets plane sections remain plane and normal to the deformed centerlines of the face-sheets. However, as the frequency increases the results calculated from these models disagree strongly with measurements. For elastic module estimation some numerical methods [10,11] were proposed. The alternative artificial intelligence method is presented in [12-14].

To model composites laminated plates, it is important to have an effective general theory for accurate evaluating the effects of transverse shear stresses on plate performance. It has long been recognized that higher-order laminated plate theories may provide an effective solution tool for accurate predicting the deformation behavior of composite laminates subjected to bending loads. It is

well known that higher-order theories, which account for transverse shear and transverse normal stresses, generally provide a reasonable compromise between accuracy and simplicity although they are usually associated with higher-order boundary conditions that are difficult to interpret in practical engineering applications.

## 2. Laminated beam modeling

Various displacement models have been developed in [2-11] by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer to investigate the phenomenon of wave propagation as well as vibrations in laminated composite plates. The present paper aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solution and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis.

Let us consider now such kinematical assumptions for symmetrical three layered plate [10, 11]:

$$U_e - \begin{cases} u = \sum_{i,k} u_{ik}^e z^i \sin k\pi x / L, & 0 < z < H, \\ w = \sum_{i,k} w_{ik}^e z^i \cos k\pi x / L, & 0 < x < L, \end{cases} \quad (1)$$

$$U_d - \begin{cases} u = \sum_{i,k} u_{ik}^d z^i \sin k\pi x / L, & H < z < H_p, \\ w = \sum_{i,k} w_{ik}^d z^i \cos k\pi x / L & 0 < x < L. \end{cases} \quad (2)$$

(here  $u_{ik}^{e,d}, w_{ik}^{e,d}$  – unknown time dependent terms). Assuming the unifrequency vibration we obtain by substitution of (1,2) into the variation low the set of linear algebraic equations for the amplitudes of vibration.

## 3. Layered beam properties in the frequency domain

The range of numerical experiments must be done to be sure that our theoretical approach is correct. We consider vibration testing of the beam for these geometrical parameters: length  $L = 0.3 \div 0.6$  m and thickness  $H = 0.0127$  m. Elastic modules are as follows:  $C_{xx} = C_{zz} = 250$  MPa,  $G = 58$  MPa, and  $C_{xz} = 40$  MPa. The frequency response function (FRF) for this beams are presented in Fig. 1–3 obtained by various range of approximations for various frequency domains (various beam length). In Fig.2 the frequencies of nonuniform sandwich-type beam are shown. From the results obtained it follows that the Euler’s beam theory overestimates frequency values. It can be observed that Tymoshenko’s beam theory also overestimates

frequency values but less than Euler’s theory. Note that, the refined theory practically coincides for the degree of approximations in longitudinal direction  $N_x > 13$  and in depth direction  $N_z > 3$ .

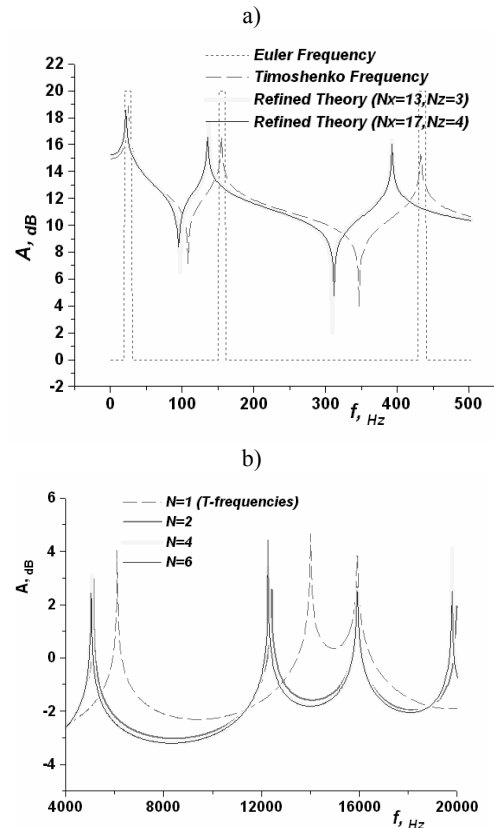


Fig. 1. Model depended FRF: (a) - for the beam with length  $L = 0.6$  m; (b) - length  $L = 0.06$  m

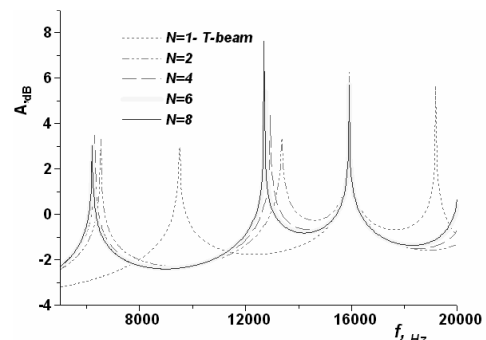


Fig. 2. Model depended FRF for nonuniform beam (length  $L = 0.06$  m).

**Damping properties in frequency domain.** Analogies theories order dependent results may be obtained also for the damping prediction. For this purpose fewer investigations are

made for damping properties investigation in frequency domain. In Fig.3. the influence of the theory order on the damping prediction accuracy is presented for the five-layered beam with intermediate damping layer.

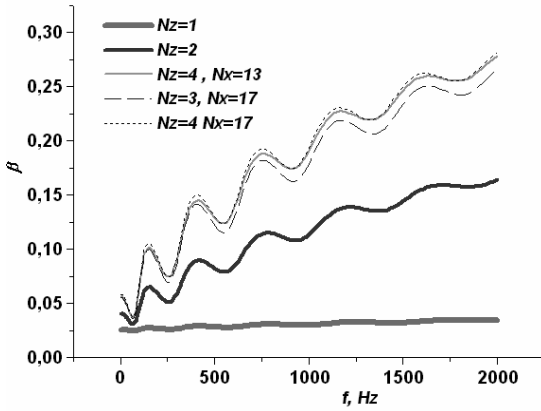


Fig. 3. Theory dependent damping prediction accuracy for the five-layered beam

In Fig.3 the frequency influence on the damping properties of sandwich panels may be seen. In Fig.4 the frequency depended damping is presented for various thicknesses damping properties distribution (only outer or core layer is damping). Two cases are under investigation: 1) damping core; 2) damping cover layer.

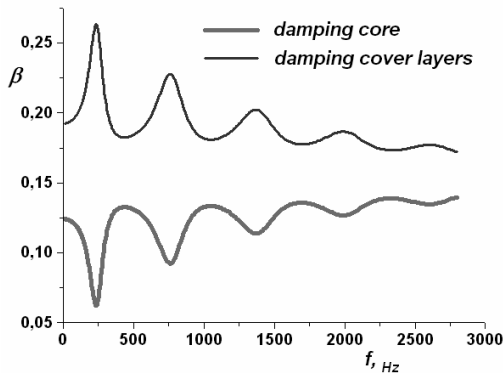


Fig. 4. Damping properties for damping core layer and for damping cover layer (L=0.6m)

In fig. 4 may be seen monotonic damping decreasing for cover damping layer and damping increasing for core damping layer. In Fig.4 may be seen damping frequency fluctuations. These fluctuations are correlated with appropriate FRF. The local dampings minimums are appropriately coincide with FRF resonance picks.

**Frequency dependent rigidity.** Let us consider now the frequency dependent characteristics of laminated beam. In Fig.5 the frequency dependent rigidity ratios  $E/E_S$  of three-layered

symmetrical beam and Euler beam with bending stiffness

$$EI_S = \int_{-H}^H E(z) z^2 dz \text{ for various layers rigidity is presented.}$$

The ratio  $E/E_S$  is  $E/E_S = f_i^2/f_{si}^2$ . (Here  $f_i$  are calculated eigen-frequencies and

$$f_{iS=H/2\pi} \sqrt{E_S g / (12\rho)} \alpha_i^2 / L^2, \alpha_1 = 1.875, \alpha_2 = 4.694,$$

known console Euler beam eigen-frequencies). Cases were discussed, when cover layer are ten times more rigid as inner layer, uniform beam and beam with ten times more rigid core.

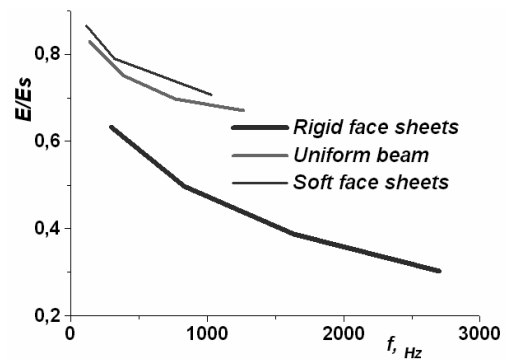


Fig. 5. Equivalent bending rigidity for sandwich

Finally let us consider the influence of damping layer position on the damping and rigid properties of the lamina. In Fig. 6. the damping properties are presented for various damping layer position. Here  $Hf$  is a thickness of rigid face layer.

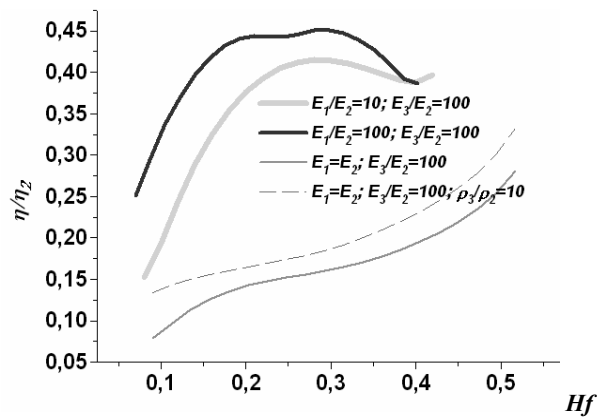


Fig. 6. Lamina properties for various damping layer position – the loss factors for the five-layered beam with damping interlayer

#### 4. Discussion

Various displacement models have been developed by considering combinations of displacement fields for in-plane and

transverse displacements inside a mathematical sub-layer to investigate the phenomenon of wave propagation as well as vibrations in laminated composite plates. Numerical evaluations obtained for wave propagation and vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibrations in laminated composite plate. The numerical method developed follows a semi-analytical approach with analytical field applied in longitudinal direction and layer-wise displacement field employed in transverse direction. The present work aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solution and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis. In [10,11] exact static solutions for composite laminates in cylindrical bending is obtained. As can be seen the order for this approximation in normal direction to the laminated plate is 3 – for normal displacement  $w$  and 2 – for in plan displacement  $u$ . If the frequency level is higher, or plate is not simply supported, but clamped (we usually in industry) or has rigid inclusions or holes, in such the case the more precision theories must be discussed. The simple example in Figs. 1-3. demonstrates low precision of theories based on assumptions to second order approximation also in the case of simply supported beam. For the every case of investigation must be solved the problem, what the theory is needed. Every theory has its limitation, among them, such as finite elements method. It is well known, that the exact solutions of elasticity theories are singular near the corners. Thus, the oscillation solutions found for stresses, for example in [15] by ABAQUES, are not obviously physically correct to the case of study.

## 5. Conclusions

The theoretical models for dynamics and damping of laminated structure have been developed. With the small number of parameters studied it predicts dynamic behavior of tested beam. Next, using this model for layered beam, higher order modeling, not only damping by shear strain in the core may be discussed, but also the damping through layer normal and curving deformation. This is important for middle and high frequency analysis of damping properties of sandwich structures. The present paper is the first attempt at proposing a novel procedure to derive stiffness parameters from forced vibrating sandwich plates. The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical or approximation method. In the future the extension of the present approach to sandwich plates will be performed in order to test various experimental conditions.

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