Archives of Materials Science and Engineering

Volume 33
Issue 2
October 2008
Pages 107-110

International Scientific Journal published monthly by the World Academy of Materials and Manufacturing Engineering

Precise tuning of the kink width in the long Josephson junction

T. Dobrowolski *
Institute of Physics, Pedagogical University, ul. Podchorążych 2, 30-084 Kraków, Poland
* Corresponding author: E-mail address: dobrow@ap.krakow.pl

Received 25.06.2008; published in revised form 01.10.2008

ABSTRACT

Purpose: The purpose of this report is to present how the external magnetic field can be employed in order to precise control the kink width in the long Josephson junction.

Design/methodology/approach: In this paper we concentrate on construction of the analytical kink solutions of the sine-Gordon model with the profile modified by the external magnetic field.

Findings: The main findings of this article are exact solutions of the sine-Gordon model which describe the squeezed or stretched kinks.

Research limitations/implications: The paper is limited to the description of the dynamics of the long Josephson junctions which are one dimensional systems with stable kink structures.

Practical implications: It is expected that the possibility of control the width of the kink will find applications in future electronic devices.

Originality/value: The main idea of the paper is to use some special magnetic field configurations to modulate (precisely) the properties of the Josephson junction.

Keywords: Long Josephson junction; Sine-Gordon model; Kink solution

METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

The Josephson effect is the phenomenon of current flow across two superconductors separated by very thin layer of insulator or normal metal. The Josephson effect was first predicted by Josephson [1] and then observed experimentally by Anderson and Rowell [2]. The effect occurs in a tunnel junction of two superconductors, in which the current across the tunnelling barrier is carried by a supercurrent of Cooper pairs. Although the Josephson junction is a two dimensional system if the transverse dimension is smaller than the Josephson length, then we are dealing with the one dimensional system called the long Josephson junction. In each of the superconductors, which form the junction, the collective ground state behaves as a single quantum particle in this sense that it is described by the wave functions:

$$\psi_1 = |\psi_0|\, e^{i\varphi_1(x,t)}, \quad \psi_2 = |\psi_0|\, e^{i\varphi_2(x,t)},$$

where $|\psi_0|$ is the amplitude. In a bulk of the superconducting materials the dominating excitation is a phase modulation of this wave functions i.e. the fields φ_1 and φ_2 are dominating degrees of freedom. This picture works until the energy is lower than the energy gap of the superconductor. The most important dynamical variable describing this system is a difference of the phases of the Landau-Ginzburg complex order parameter of the two superconductors comprising the junction. The dynamics of the phase difference between the two superconducting electrodes $\phi(x,t) = \varphi_2(x,t) - \varphi_1(x,t)$ in this system is described by the one dimensional sine-Gordon equation. In the presence of an external magnetic field this equation, in dimensionless coordinates, takes the form:

$$\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial t^2}=\sin\phi+\frac{\partial H_{ext}}{\partial x}, \tag{1}$$

where H_{ext} is the normalized magnetic field. In the above equation we neglected the quasiparticle current and assumed the zero bias current. The connection of the dimensionless coordinates (x,t) with the original space-time coordinates (X,T) is the following:

$$x=\lambda_J^{-1}X,\ \ t=\omega_P T, \tag{2}$$

where

$$\lambda_J=\sqrt{\frac{\Phi_0}{2\pi L^* J_c}}, \tag{3}$$

is the Josephson length and

$$\omega_P=\sqrt{\frac{2\pi J_c}{\Phi_0 C^*}}, \tag{4}$$

is the plasma frequency. Here J_c denotes the critical current density, $\Phi_0=h/(2e)$ is the flux-quanta, C^* is the specific capacitance of the barrier and L^* denotes the sheet inductance. The equation of motion (1), in the homogenous external magnetic field $(H_{ext}=const)$, posses the kink solution

$$\phi_K(x)=4\arctan(e^{x-x_0}), \tag{5}$$

that satisfies the boundary conditions $\phi(x=-\infty)=0$ and $\phi(x=\infty)=2\pi$ [3-5]. If we interchange the boundary conditions and change the sign before the expression $(x-x_0)$ then we also obtain the antikink solution. Kinks are representatives of quite broad class of solutions called topological solitons. The solutions of this type occur in modelling of many condensed matter systems. They appear in description of ferromagnetism, superconductivity, superfluidity, liquid crystals and many other systems where they have a form of domain walls [6], kinks [7], textures [8-9], and vortices [10]. The structures of this type are created by application of special external conditions or they are created spontaneously during the phase transitions in the condensed matter systems. The statistical description of creation process is presented in a series of papers [11-14]. Besides the statistical approach there exists a detailed analytical and numerical descriptions of the dynamical mechanisms which stands behind the statistical analysis [15-16]. The experimental verification and motivation of these studies can be found in papers [17-20]. In this paper we concentrate on construction of the analytical kink solutions of the sine-Gordon model with the profile modified by the external magnetic field.

2. Kink solutions of arbitrary width

Let us consider the kink in the non-homogenous magnetic field of the form

$$H_{ext}=H_0\text{sech}[A(x-x_0)], \tag{6}$$

where H_0 is the field amplitude and A is (temporary) an arbitrary constant. We also presume particular ansatz for the kink solution in the magnetic field (6)

$$\phi_K(x)=4\arctan(e^{A(x-x_0)}), \tag{7}$$

where A is still an arbitrary constant. If we put above ansatz and the magnetic field (6) into the equation of motion (1) then we obtain the equation that fixes the constant A i.e.

$$2A^2-H_0A-2=0. \tag{8}$$

The solutions

$$A_\pm=\frac{H_0}{4}\pm\sqrt{\left(\frac{H_0}{4}\right)^2+1}, \tag{9}$$

together with the magnetic field (6) and the kink ansatz (7) resolves the problem of tuning of the kink width in the long Josephson junction.

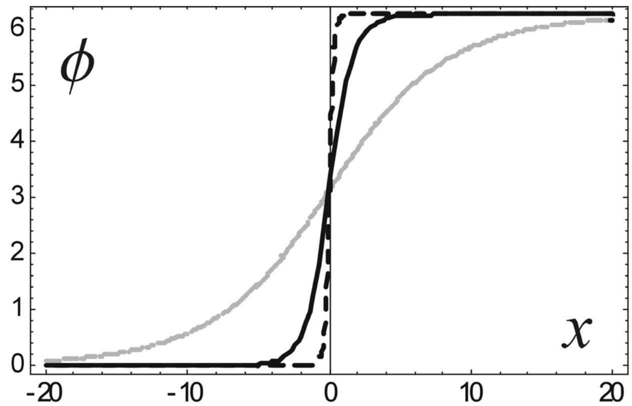

Fig. 1. The kink profile in the zero external magnetic field $H_0=0$ (solid dark line). In the case of $H_0=-10$ the kink is stretched (solid grey line) and in the case of $H_0=10$ the kink is squeezed (dashed line)

Let us notice that always $A_+>0$ and also $A_-<0$ and therefore A_+ corresponds to the tuned kink and A_- to the tuned antikink. Depending on the sign of the constant H_0 the kink can be arbitrary squeezed or stretched. If H_0 is positive then the kink is squeezed and for negative H_0 the kink is stretched. In case of the antikink an inverse effect occurs. Moreover in the same way we can tune the width of the stationary kink moving with the speed v. In this case the expression $(x-x_0)$ in the formulas (6) and (7) have to be replaced by (Fig. 2) $(x-\mathrm{v}t-x_0)$.

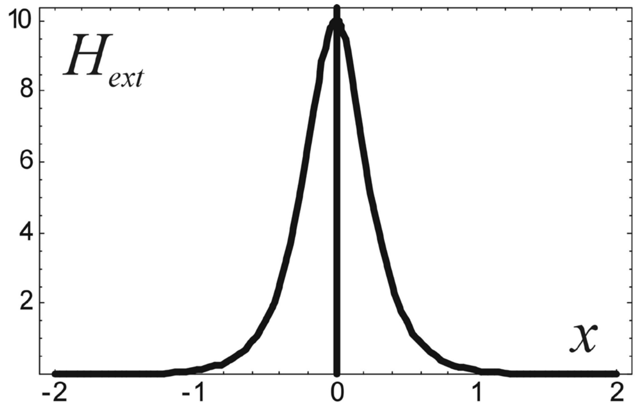

Fig. 2. The external magnetic field of amplitude $H_0=10$ applied in order to obtain the squeezed kink represented by the dashed line in Figure 1

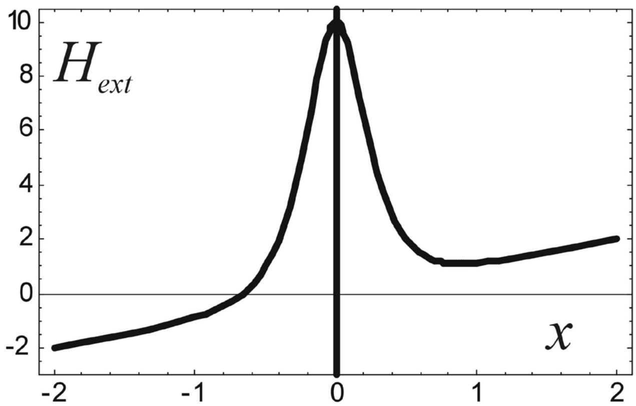

Fig. 3. The external magnetic field of amplitude $H_0=10$ in the model with the external bias current $\gamma=1$, applied in order to obtain the squeezed kink represented by the dashed line in Figure 1

The similar considerations can be performed in the system with nonzero quasiparticle and external bias current $\gamma=J/J_c$. The equation of motion in this more general case is the following:

$$\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial t^2}=\sin\phi+\frac{\partial H_{ext}}{\partial x}-\gamma+\alpha\frac{\partial\phi}{\partial t}, \qquad (10)$$

where α is a quasiparticle damping. Due to lack of the Lorentz invariance the kink solutions of this model are static. The same procedure as applied before produces identical tuned kink (Fig. 3) solution (7, 9) but this time we have to apply the external magnetic field of the form

$$H_{ext}=\gamma x+H_0\mathrm{sech}[A_\pm(x-x_0)], \qquad (11)$$

where $A_\pm$ is given in the formula (7).

3. Conclusions

In the article presented above we showed how to prepare the external conditions in order to control the width of the kinks in the long Josephson junction. We expect that the possibility of precise control of the parameters of the Josephson junction will find application in a future electronic devices. The validity of the proceeded by us considerations is limited by the validity of the dimensional reduction of the system and therefore the validity of the used by us equation is limited to the junctions of width smaller than the Josephson length. Only in this case the junction can be treated as a one dimensional system. In this limit, the currents perpendicular to the longitudal axis can be neglected. Moreover, this approximation requires that the determinant of the transformation from the junction coordinates to the laboratory coordinates is close to unity, which essentially limits this approximation to junctions with small curvature along the longitudal axis.

Acknowledgements

I would like to thank the members of the Department of Dielectrics and Biophysics AP in Cracow for encouraging me to prepare this paper.

References

[1] B.D. Josephson, Possible new effects in superconductive tunnelling, Physics Letters 1 (1962) 251-253.

[2] P.W. Anderson, J.M. Rowell, Probable Observation of the Josephson Superconducting Tunnelling Effect, Physical Review Letters 10 (1963) 230-232.

[3] M.J. Ablowitz, P.A. Clarkson, Solitons, nonlinear evolution equations and inverse scattering, Cambridge University Press, Cambridge, 1999.

[4] L.A. Ferreira, B. Piette, W.J. Zakrzewski, Wobbles and other kink-breather solutions of the sine-Gordon model, Physical Review E 77 (2008) 036613-036622.

[5] L.A. Ferreira, B. Piette, W.J. Zakrzewski, Dynamics of the topological structures in inhomogeneous media, Journal of Physics A (2007).
[6] C. Hakan Gur, I. Cam, Investigation of as-quenched and tempered commercial steels by Magnetic Barkhausen Noise method, International Journal of Microstructure and Materials Properties 1 (2006) 208-218.
[7] S.H. Lin, J. Pan, Fatigue life prediction for spot welds in coach-peel and lap-shear specimens with consideration of kinked crack behaviour, International Journal of Materials and Product Technology 20 (2004) 31-50.
[8] S. Tanaka, K. Yasuda, Y. Matsuo, Influence of microstructure on modal work-of-fracture of carbon fibre/pitch-derived carbon composites, International Journal of Materials and Product Technology 16 (2001) 171-179.
[9] K. Rodak, T. Goryczka, The microstructural characteristics of Al processed using severe plastic deformation procedures, International Journal of Computational Materials Science and Surface Engineering 1 (2007) 585-593.
[10] J. Hemanth, Effect of microstructure on mechanical behaviour of chilled Pb/Sb/Sn/As Alloy-Al2SiO5 (Kaolinite) metal matrix composites, International Journal of Microstructure and Materials Properties 1 (2005) 74-87.
[11] T. Dobrowolski, Kink production in the presence of impurities, Physical Review E 65 (2002) 036136-036141.
[12] T. Dobrowolski, Kink production in the presence of random distributed impurities, Physical Review E 65 (2002) 046133-046137.
[13] W. H. Zurek, Cosmological experiments in superfluid Helium, Nature 317 (1985) 505 – 508.
[14] T. Dobrowolski, Hedgehog production in spatially correlated noise, European Physical Journal B 29 (2002) 269-271.
[15] T. Dobrowolski, Kinks of arbitrary width, Physical Review E 66 (2002) 066112-066118.
[16] P. Tatrocki, T. Dobrowolski, Defect production behind the shock wave front of an inhomogeneous quench, Physical Review E 69 (2004) 016209-016215.
[17] A. Maniv, E. Polturak, G Koren, Observation of magnetic flux generated spontaneously during a rapid quench of superconducting films, Physical Review Letters 91 (2003) 197001-197004.
[18] T. Riste, L. Dobrzyński, Nematic-Isotropic Transition: Thermal Hysteresis and Magnetic Field Effects, Physical Review Letters 74 (1995) 2737-2739.
[19] M.J. Bowick, L. Chandar, E.A. Schiff, A.M. Srivastava, The cosmological Kibble mechanism in the laboratory: string formation in liquid crystals, Science 263 (1994) 943-945.
[20] I. Chuang, R. Durrer, N. Turok, B. Yurke, Cosmology in the laboratory: defect dynamics in liquid crystals, Science 251 (1991) 1336-1342.