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An influence of the curvature on the kink dynamics in the spherical Josephson junction

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ABSTRACT

Purpose: Purpose of this report is to present the impact of the spherical geometry on the kink motion in large area Josephson junction.

Design/methodology/approach: The effective Lagrangian method is used in order to obtain the dynamics of the sine-Gordon kink on a curved manifold.

Findings: The main finding of this article is an effective potential that describes the influence of the curvature on the kink motion in the spherical Josephson junction.

Research limitations/implications: The paper is limited to the description of the dynamics of kinks that move with non relativistic speeds in narrow Josephson junctions.

Practical implications: It seems that junctions with appropriate geometry will find applications in future electronic devices. It is expected that curved Josephson junctions can be used in order to store a binary data.

Originality/value: The main idea of the paper is to use a Riemann geometry in order to describe the influence of the curvature on the kink motion in the spherical junction.

Keywords: Josephson junction; Sine-Gordon model; Kink solution

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

The phenomenon of current flow across junction made of two superconductors separated by very thin layer of insulator or normal metal was first described by Josephson [1]. The positive experimental verification of the Josephson idea was performed by Anderson and Rowell [2]. In a junction of two superconductors the current across the tunnelling barrier is carried by overcurrent of Cooper pairs. In each of the superconductors, which form the junction, the collective ground state behaves as a single quantum particle and it is described by the many particle wave function of constant amplitude $|\psi_0|$:

$$\psi_1 = |\psi_0| \, e^{i\varphi_1(\vec{x},t)}, \quad \psi_2 = |\psi_0| \, e^{i\varphi_2(\vec{x},t)}.$$

In a bulk of the superconducting materials the dominating excitation is a phase modulation of these wave functions i.e. φ_1 and φ_2. This picture works until the energy is lower than the energy gap of the superconductor. The most important dynamical variable that describes this system is a difference of the phases of the Landau-Ginzburg complex order parameters of the two superconductors comprising the junction. The dynamics of the phase difference between superconducting electrodes $\phi(\vec{x},t)=\varphi_2(\vec{x},t)-\varphi_1(\vec{x},t)$ in this system, in the absence of the quasiparticle and bias currents, is described by the sine-Gordon model. Additionally we assume zero external magnetic field. This model is defined by the Lagrangian density:

$$L=\frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)-(1-\cos\phi), \quad (1)$$

where $\eta^{\mu\nu}=diag(+1,-1,-1,-1)$ is Minkowski metric in dimensionless Cartesian coordinates $(x^\mu)=(t,x,y,z)$. The connection of these coordinates with original space-time coordinates (T,X,Y,Z) is the following:

$$t=\omega_P T,\ x=\lambda_J^{-1}X,\ \ y=\lambda_J^{-1}Y,\ \ z=\lambda_J^{-1}Z, \quad (2)$$

where

$$\omega_P=\sqrt{\frac{2\pi J_c}{\Phi_0 C^*}}, \quad (3)$$

is plasma frequency and

$$\lambda_J=\sqrt{\frac{\Phi_0}{2\pi L^* J_c}}, \quad (4)$$

is Josephson length. Here J_c denotes the critical current density, $\Phi_0=h/(2e)$ is the flux-quanta, C^* is the specific capacitance of the barrier and L^* denotes the sheet inductance. The field equation for Lagrangian density (1), possesses the kink solution [3-5]. Kinks are representatives of much broader class of solutions called topological solitons. The solutions of this type occur in studies on many condensed matter systems. Topological solitons appear in description of ferromagnetism, superfluidity, superconductivity, liquid crystals and many other systems where they have a form of kinks [6], domain walls [7], textures [8-9], and vortices [10]. The structures of this type are created during the phase transitions in the condensed matter systems. The statistical description of creation process is presented in a series of papers [11-14]. The experimental verification and motivation of these studies can be found in papers [15-20]. Besides the statistical approach there exists a detailed analytical and numerical description of the dynamical mechanisms which stands behind the statistical analysis [21-24]. In this paper we analyse an influence of the curvature on the propagation of the kink along with the junction.

2. An influence of the curvature on the kink motion

In order to describe the dynamics of the kink in the curved junction we introduce a special curved coordinates $(\xi^\alpha)=(\xi^1,\xi^2,\xi^3)=(s,\sigma,\xi)$. Two first coordinates $(\sigma^a)=(\sigma^1,\sigma^2)=(s,\sigma)$ parameterise the surface of the junction i.e. the surface of the insulator that separates the superconducting electrodes. These parameters we choose as two first curved coordinates that parameterise the space. The last curved coordinate ξ we choose as a normal coordinate to the surface of the junction. The position of the points of the junction we identify by the vector $\vec{X}(s,\sigma)$. The implicit relation between the Cartesian (x^i) and curved (ξ^α) coordinates have the form:

$$\vec{x}=\vec{X}(s,\sigma)+\xi\,\vec{n}(s,\sigma) \quad (5)$$

where $\vec{n}$ is vector normal to the surface (see Fig.1).

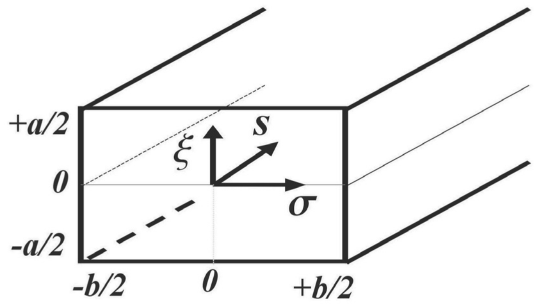

Fig. 1. Two coordinates s and σ parameterise a surface of the junction. The last coordinate ξ indicates the direction normal to the junction

The reduction of the model (1) to two dimensions is achieved if one presumes that the solutions of the model do not depend on normal variable to the junction. The kinetic energy in the reduced theory has a form (see [25] for details):

$$T=a\int_0^l ds\int_{-b/2}^{+b/2}d\sigma\sqrt{g}\left(1+\tfrac{a^2}{24}\boldsymbol{R}\right)(\partial_t\phi)^2, \quad (6)$$

where $\boldsymbol{R}$ is scalar of Riemann curvature and g is determinant of the metric induced on the surface of the junction. For small curvatures one can obtain also the general formula for the potential energy:

$$U \approx a\int_0^l ds \int_{-b/2}^{+b/2} d\sigma\sqrt{g}\left\{\left[\tfrac{1}{2}\left(1-\tfrac{a^2}{24}\mathbf{R}\right)g^{ab} + \right.\right. \tag{7}$$

$$\left.\left. + \tfrac{a^2}{24}K^{ac}K^{b}{}_{c}\right](\partial_a\phi)(\partial_b\phi) + \left(1+\tfrac{a^2}{24}\boldsymbol{R}\right)(1-\cos\phi)\right\},$$

where K_{ab} denote the external curvatures and g_{ab} is metric induced on the surface of the junction. In the above formula we use Einstein convention with respect to repeating indices. The effective description of the kink dynamics we achieve in the framework of the collective coordinate method. We adopt the following kink ansatz:

$$\phi = 4\arctan(e^{(s-S(t))}), \tag{8}$$

where the function $S(t)$ indicate a position of the kink in an arbitrary instant of time. This ansatz means that the shape of the kink front is not modified during its motion through the junction. This kind of motion can be achieved, for example, in a narrow and slightly curved Josephson junction.

The effective kinetic and potential energy that describes the motion of the kink along the junction can be obtained by integration the expressions (6, 7) over the spatial variables. First we have to fix the geometry of the junction. In this article the junction has a form of the cylinders (for parameter $s \in [0,s_1] \bigcup [s_2,l]$) joined by the spherical surface for $s \in [s_1,s_2]$ - see Fig.2.

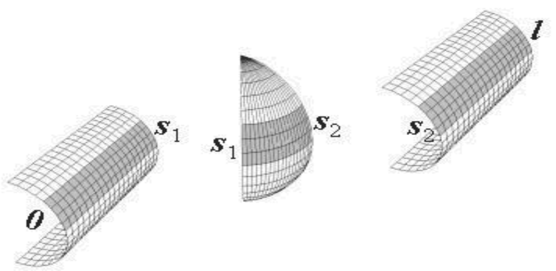

Fig. 2. Symbolic picture of the curved junction. Grey areas represents the subsequent sections of the junction

The cylinders are parameterised by the parameter s chosen along the symmetry axis and therefore the radius vector has a form:

$$\vec{X}(s,\sigma) = r\cos\tfrac{\sigma}{r}\vec{e}_x + r\sin\tfrac{\sigma}{r}\vec{e}_y + s\,\vec{e}_z \tag{9}$$

The metric tensor, external curvatures and the curvature scalar in the above parameterisation are the following:

$$g_{ab} = \delta_{ab},\ K_{ss} = K_{s\sigma} = 0,\ K_{\sigma\sigma} = -\tfrac{1}{r},\ \boldsymbol{R} = 0 \tag{10}$$

The effective kinetic energy of the kink in the cylindrical sections follows directly from the formula (6)

$$T_{cylinder} = 2ab[J(S,0,s_1) + J(S;s_2,l)]\dot{S}^2, \tag{11}$$

where $J(S;x,y)$ is almost step function i.e. it is constant (and equal two) in the interval $[x,y]$ and zero outside (see Fig.3).

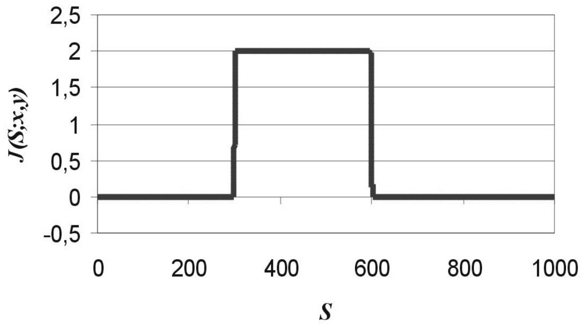

Fig. 3. Step-like function *J(S;x,y)* for *x*=300 and *y*=600. We presumed the total length of the junction *l*=1000

The effective potential energy in the cylindrical sections is obtained directly from the formula (7)

$$U_{cylinder} = 4ab[J(S;0,s_1) + J(S;s_2,l)] \tag{12}$$

In the spherical part of the junction, i.e. for parameter s belonging to the interval $[s_1,s_2]$, the radius vector has the form:

$$\vec{X} = r\cos\tfrac{\sigma}{r}\cos\tfrac{s}{r}\vec{e}_x + r\cos\tfrac{\sigma}{r}\sin\tfrac{s}{r}\vec{e}_y + r\sin\tfrac{\sigma}{r}\vec{e}_z\,. \tag{13}$$

In the above parameterisation the metric induced on the spherical part of the surface of the junction is represented by diagonal matrix

$$[g_{ab}] = \begin{bmatrix} \cos^2\frac{\sigma}{r} & 0 \\ 0 & 1 \end{bmatrix}. \tag{14}$$

The geometry of this section is nontrivial in this sense that it has not only nontrivial embedding but it also is internally curved

$$K_{ss} = -\tfrac{1}{r}\cos^2\tfrac{\sigma}{r},\ K_{\sigma\sigma} = -\tfrac{1}{r},\ K_{s\sigma} = 0,\ \boldsymbol{R} = \tfrac{2}{r^2} \tag{15}$$

The effective kinetic energy of the kink in this section has the form:

$$T_{sphere} = 4aR\sin(\tfrac{b}{2r})\left(1+\tfrac{a^2}{12r^2}\right)J(S;s_1,s_2)\dot{S}^2 \tag{16}$$

If we assume that the transverse direction of the junction is much smaller than the radius of the spherical section $r >> b$ then the kinetic energy became linear in b.

$$T_{sphere} \approx 2ab\left(1+\tfrac{a^2}{12r^2}\right)J(S;s_1,s_2)\dot{S}^2 \tag{17}$$

The effective potential energy of the kink has the form

$$U_{sphere} = 4ar\left[\sin\left(\tfrac{b}{2r}\right)+\ln\left|\frac{1+tg\left(\frac{b}{4r}\right)}{1-tg\left(\frac{b}{4r}\right)}\right|\right]J(S;s_1,s_2) \tag{18}$$

In the large radius regime $(r >> b)$ this expression simplifies significantly:

$$U_{sphere} \approx 2ab\left(2+\tfrac{b^2}{24r^2}\right)J(S;s_1,s_2) \tag{19}$$

Finally, if we use equations (11, 17) we obtain the unified expression that gives kinetic energy of the kink in the curved Josephson junction.

$$T \approx \frac{1}{2}\left[8ab + ab\frac{a^2}{3r^2}J(S;s_1,s_2)\right]\dot{S}^2 \tag{20}$$

This expression suggests that the effective mass of the kink in the cylindrical sections $(J = 0)$ is smaller than in the spherically deformed element of the junction $(J = 2)$. The unified expression for the potential energy in the whole junction is obtained from the formulas (12, 19).

$$U \approx 8ab + ab\frac{b^2}{12r^2}J(S;s_1,s_2) = 8ab + \Delta U \tag{21}$$

The properties of the function $J(S;x,y)$ show the existence of the energy barrier in the spherical section of the junction.

$$\Delta U \approx \frac{ab^3}{6r^2} \tag{22}$$

This energy barrier can be used in order to stop the kink in its movement along the junction.

3. Conclusions

The recent progress in micro-technology made it possible to fabricate Josephson junctions with complicated geometry. It was shown that curvature is responsible for existence of the potential barriers in the junction. A size and shape of the barrier is directly determined by the geometry of the junction. From technical point of view these barriers can be arranged in order to form the hole configuration. On the other hand the potential holes could find future applications in electronic devices in order to store the binary data. This application is related to possibility of confining the fluxons in the holes. The control of the trapping process may be achieved by introducing the external magnetic field and small damping to the system. The external magnetic field can be used in order to push the fluxon in to the potential hole. On the other hand the damping could help to decrease the kinetic energy of the fluxon. In this paper we considered the junction that consists of two cylindrical and one spherical sections. It was showed that the spherical section of this junction works as a potential barrier. Moreover we established the direct connection of the radius of the spherical section with the height of the barrier and other parameters of the junction (22). Finally, we would like to underline that proceeded by us considerations have several limitations. First, we used approximation of low curvatures that means that the curvature radius is much larger than the thickness and the width of the junction i.e. $r >> b >> a$. Second, we assume that the kink front is not disturbed during its motion through the junction. One could expect that this assumption is satisfied in sufficiently narrow junctions. In addition, we used the collective coordinate method only with position variable that limits its validity to non relativistic speeds.

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