



# Acceleration of charged particles in laser beam

**M.J. Małachowski\*, A. Dubik**

Education Department, Technical University of Radom named after K. Pułaski,  
ul. J. Malczewskiego 20A, 26-600 Radom, Poland

\* Corresponding author: E-mail address: malachowski.m.j@interia.pl

Received 26.09.2009; published in revised form 01.12.2009

## ABSTRACT

**Purpose:** The aim of this paper was to find parameters of the laser and maser beams in numerical ways with additionally applied external static axial magnetic field which satisfies the proper conditions for charged particle acceleration.

**Design/methodology/approach:** The set acceleration was designed in order to obtain the possible high kinetic energy of the charged particles in the controllable manner. This was achieved applying a circularly polarized high intensity laser beam and a static axial magnetic field, both acting on the particle during the proper period.

**Findings:** The quantitative illustrations of the calculation results, in a graphical form enabled to discuss the impact of many parameters on the acceleration process of the electrons and protons. We have found the impact of the Doppler Effect on the acceleration process to be significant. Increase in laser or maser beam intensity results in particle's energy increase and its trajectory dimension. However, increase in external magnetic field results in shrinking of the helical trajectories. It enables to keep the particle inside the laser beam.

**Research limitations/implications:** Limits in the energy of accelerated particles arise from the limits in up-to-date available laser beam energy and the beam diameters.

**Originality/value:** The authors show the parameters of the circularly polarized laser beam which should be satisfied in order to obtain the desired energy of the accelerated particles. The influence of the magnetic field strength is also shown.

**Keywords:** Acceleration of charged particles; Laser; Maser; Relativistic dynamics

**Reference to this paper should be given in the following way:**

M.J. Małachowski, A. Dubik, Acceleration of charged particles in laser beam, Archives of Materials Science and Engineering 40/2 (2009) 98-103.

## MATERIALS MANUFACTURING AND PROCESSING

### 1. Introduction

The interest in practical development of accelerating devices for charged particles recently has rapidly grown. The elaboration of new theoretical models and many successful experiments resulting in gaining by the charged particles the relativistic energies gave an additional impulse for these studies. This direction of investigation is so interesting because of considerable progress in construction of lasers and masers which were used to

perform verification of the new models of particle acceleration. No doubt these accelerating devices begin to be alternative accelerators used to date.

There is some evidence of existing accelerating devices capable of generating narrowly shaped beams of charged particles moving with relativistic velocity (electrons, positrons, and also particles of higher masses as protons and ions). Further progress in this subject can make an essential influence in many areas of technology and science [1, 2, 3, 4].

During the last few years, a possibility of particles acceleration has been shown as a result of their interaction with the laser or maser beams of different polarization [5, 6] and additionally, static or pulsating magnetic or electric fields applied having different intensity and direction [7, 8]. Now there are accessible lasers showing radiation power density of the level  $10^{22}$  W/cm<sup>2</sup>, which corresponds to the electric field intensity amplitude  $10^{14}$  V/m. This has triggered conduction of many new researches [9, 10, 11]. It was found that the chirping frequency plays an important role to enhance the electron energy if the laser is tightly focused [12]. There are known presentations of new types of charged particle accelerators, as an example we can give the method of acceleration known as a dielectric wall accelerator [13].

The main purpose of this paper is to show the electron and proton trajectories under various parameters of electromagnetic and magnetic fields of laser beam and to find the conditions at which the particles can be accelerated to enlarge energies under interaction with the laser radiation and the static magnetic field. Another purpose is to indicate the significance of the Doppler Effect in the acceleration process.

## 2. Equations describing the motion of the charged particles in a laser beam and an axial static magnetic field

We assume that the laser produces continuous, monochromatic, coherent, and circularly polarized and plane wave propagated in vacuum and additionally applied in the same or contrary directed static axial magnetic field. The aim of this work is to find the impact of this complicated electromagnetic field on the trajectory and kinetic energy of charged particle in the lossless conditions especially in the relativistic region.

Dynamical relativistic equation and the continuous equation of normalized energy  $\gamma$  have the following form:

$$\begin{aligned} \frac{d\vec{p}}{dt} &= q\vec{E} + q[\vec{V} \times (\vec{B} + \vec{B}_z)] \\ \frac{d\gamma}{dt} &= \frac{q}{m_0 c^2} \vec{V} \cdot \vec{E} \end{aligned} \quad (1)$$

where  $p$ ,  $q$  and  $m_0$  are the momentum, the charge and the rest mass of the particle,  $E$ ,  $B$  and  $c$  are the electric field intensity, the magnetic field induction and the velocity of electromagnetic wave, respectively,  $V$  is the particle's velocity and  $B_z$  is the external static magnetic induction along the  $z$  coordinate. And

$$\begin{aligned} E_x &= E_0 \sin \phi, & E_y &= E_0 \cos \phi \\ B_x &= \frac{E_0}{c} \cos \phi, & B_y &= \frac{E_0}{c} \sin \phi, \\ B_z &= \pm a \frac{E_0}{c} \end{aligned}$$

$$\begin{aligned} \gamma &= (1 - \beta^2)^{-\frac{1}{2}}, & \vec{\beta} &= \frac{\vec{V}}{c}, & \vec{p} &= \gamma \cdot m_0 c \vec{\beta}, \\ \beta^2 &= \beta_x^2 + \beta_y^2 + \beta_z^2, & \beta_{x,y,z} &= \frac{V_{x,y,z}}{c} \end{aligned} \quad (2)$$

where  $E_0$  is the amplitude of electric field,  $a$  is the parameter defining the field  $B_z$  directed along the  $z$  coordinate. In a complex system consisting of an electromagnetic field and a static axial magnetic field the following vectors are essential

$$\vec{E} = [E_x, E_y, 0], \quad \vec{B} = [-B_x, B_y, B_z]$$

and

$$\begin{aligned} \vec{p} &= m_0 \gamma V_x \vec{i} + m_0 \gamma V_y \vec{j} + m_0 \gamma V_z \vec{k} \\ \frac{d\vec{p}}{dt} &= \vec{i} m_0 c \frac{d\gamma \beta_x}{dt} + \vec{j} m_0 c \frac{d\gamma \beta_y}{dt} + \vec{k} m_0 c \frac{d\gamma \beta_z}{dt} \end{aligned} \quad (3)$$

On the basis of the above remarks we shall derive equations describing dynamics of a charged particle in a relativistic case. Inserting Eq. (2) in equations (1) describing dynamics of a particle for  $B_z$  defined by the parameter  $a > 0$  we get the equations of motion in the form

$$\begin{aligned} \frac{d\gamma \beta_x}{dt} &= -\omega \alpha (1 - \beta_z) \sin \phi - \omega \alpha \beta_y, \\ \frac{d\gamma \beta_y}{dt} &= -\omega \alpha (1 - \beta_z) \cos \phi + \omega \alpha \beta_x, \\ \frac{d\gamma \beta_z}{dt} &= -\omega \alpha \beta_x \sin \phi - \omega \alpha \beta_y \cos \phi, \\ \frac{d\gamma}{dt} &= -\omega \alpha \beta_x \sin \phi - \omega \alpha \beta_y \cos \phi \end{aligned} \quad (4)$$

with

$$\phi = \omega_l \sqrt{\frac{1 - \beta_z}{1 + \beta_z}} \left[ t - \frac{z(t)}{c} \right], \quad (5)$$

where  $\phi$  is the phase,  $\omega$  and  $\omega_l$  are the frequencies of laser wave in particles frame of reference and laser frame of reference, respectively.

As it can be seen from relation (5), the Doppler Effect is included. This effect was found to be of significant importance in the acceleration process.

Finally the total reduced velocity  $\beta$  of the particle can be calculated from relation (2). The kinetic relativistic energy was calculated from the following relativistic formula

$$E_k = m_0 c^2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \quad (6)$$

### 3. Presentation of simulation results in a graphical form

The original equations (4) were used to calculate the time dependence of the particle's trajectory parameters and its velocity. The kinetic energy of the particle was calculated using equation (6). In order to perform the calculations, the Runge-Kutta Method procedure has been applied. The below attached figures show the calculation results obtained using this method. The acceleration process of an electron and a proton was numerically analyzed. We selected two values of laser wavelengths: 1 and 10  $\mu\text{m}$  and 1 mm wavelength lying in a maser radiation range. The laser or maser intensities were defined by amplitude of their electrical field. We have chosen  $10^9$ ,  $10^{10}$  and  $10^{11}$  V/m, the values rather high but already achievable. As the initial conditions, the  $z$  compound of velocity has been chosen as  $c/1000$ , while the remaining velocity and position compounds were zero in all presented cases.

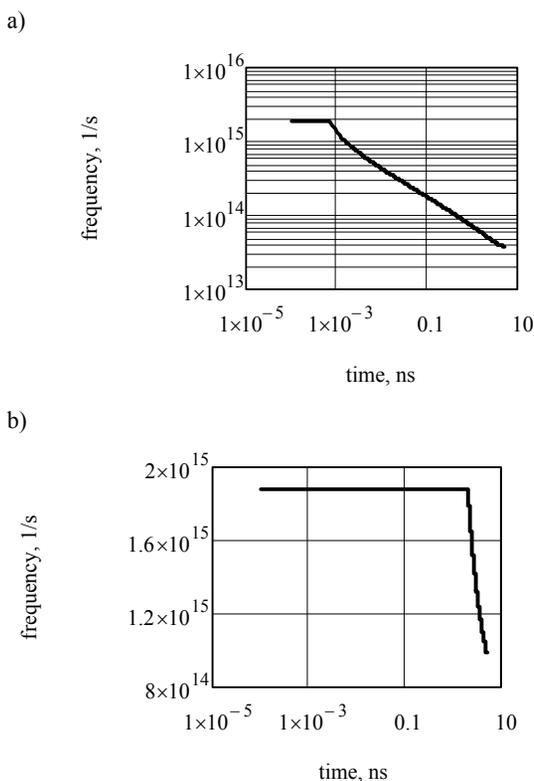


Fig. 1. Variation in time of laser radiation frequency 'seen' by an accelerated electron (a) and proton (b) as a result of interaction with laser beam. Amplitude of electric field of electromagnetic wave  $E_0 = 10^{10}$  V/m, the wavelength  $\lambda = 1 \mu\text{m}$  and the induction of static magnetic field  $B_z = 260$  T for an electron and  $-200$  T for a proton

The main difference between an electron and a proton acceleration process lays in the rate at which velocity is gained. The electron velocity almost immediately approaches the limit of the light velocity in vacuum, while the velocity of proton advances much slower. This results in the difference in the laser radiation frequencies 'seen' by these particles. Fig. 1 shows the change in time of radiation frequency in the particle's frame of reference. As it can be seen the laser frequency for an electron starts to decrease nearly from the beginning of the motion, while for a proton it begins to decrease later on, when the velocity approaches the relativistic region. The lower laser frequency means the radiation cycle acting on the particle is longer and a particle needs a smaller number of cycles to gain the expected kinetic energy compared to the situation when the Doppler Effect can be omitted. The moment of the decrease in the laser frequency 'seen' by a particle, means velocity of the particle entered in to the relativistic region. When the process of acceleration starts, the particle moves along the helical trajectory advancing in the direction of laser beam axis. For the acceleration process it is better the radius of trajectory should be no larger than the beam radius. The former can be controlled by the intensity of the magnetic field. The larger the field is, the smaller the radius is. When the Doppler Effect is considered, the intensity of the static magnetic field was found to exert only little impact on the particles velocity. In this paper we decided not to discuss this effect. In order to understand the particle motion better, we have also plotted the projection of the trajectory on to the  $(x, y)$  plane shown in Fig. 2. Having the trajectory projection and the  $z(t)$  dependence (Fig. 3) we can imagine the helical trajectory in the three dimension space.

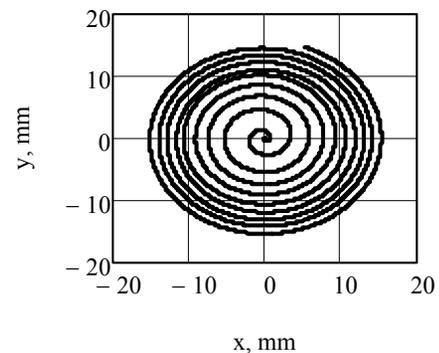


Fig. 2. The projection on to the  $(x, y)$  plane of the trajectory of accelerated proton during 10 ns as a result of interaction with laser beam and static magnetic field. Amplitude of electric field of electromagnetic wave  $E_0 = 10^{10}$  V/m, the wavelength  $\lambda = 1 \mu\text{m}$  and the induction of static magnetic field  $B_z = -300$  T

Fig. 3 shows the position of the particle along the laser beam at each moment of acceleration. It can be seen that for electron this position approximately can be found simply by the product of light velocity and the acceleration time, while for the proton it is not the case, since its velocity approaches vicinity of  $c$  much later.

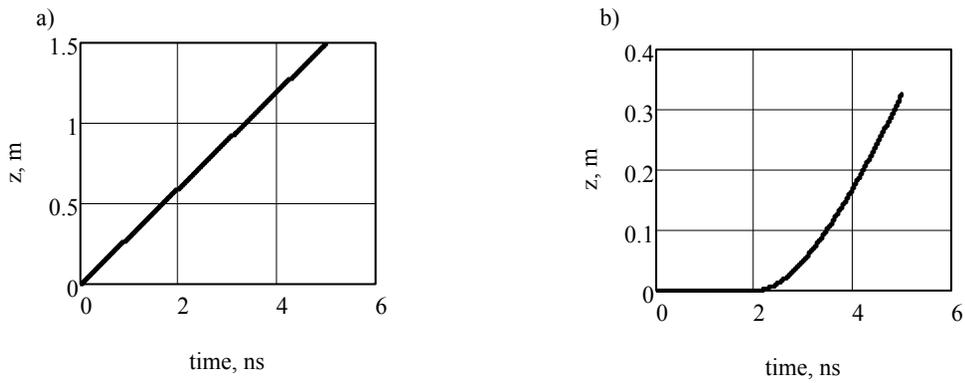


Fig. 3. Variation in time of the accelerated electron position along the  $z$  axis (a) and proton (b) as a result of interaction with laser beam. Amplitude of electric field of electromagnetic wave  $E_0 = 10^{10}$  V/m, the wavelength  $\lambda = 1 \mu\text{m}$  and the induction of static magnetic field  $B_z = 260$  T for the electron and  $-200$  T for the proton

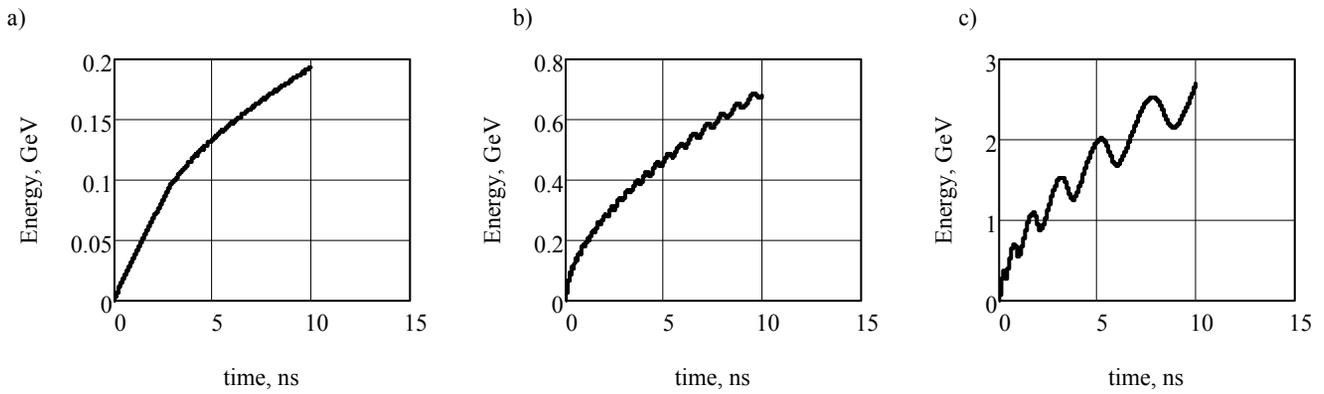


Fig. 4. Variation in time of accelerated electron kinetic energy gained as a result of interaction with laser beam of wavelength  $\lambda = 10 \mu\text{m}$ . Amplitude of electric field of electromagnetic wave  $E_0 = 10^9$  V/m (a),  $10^{10}$  V/m (b),  $10^{11}$  V/m (c) and the induction of static magnetic field  $B_z = 62$  T (a) and  $50$  T (b, c)

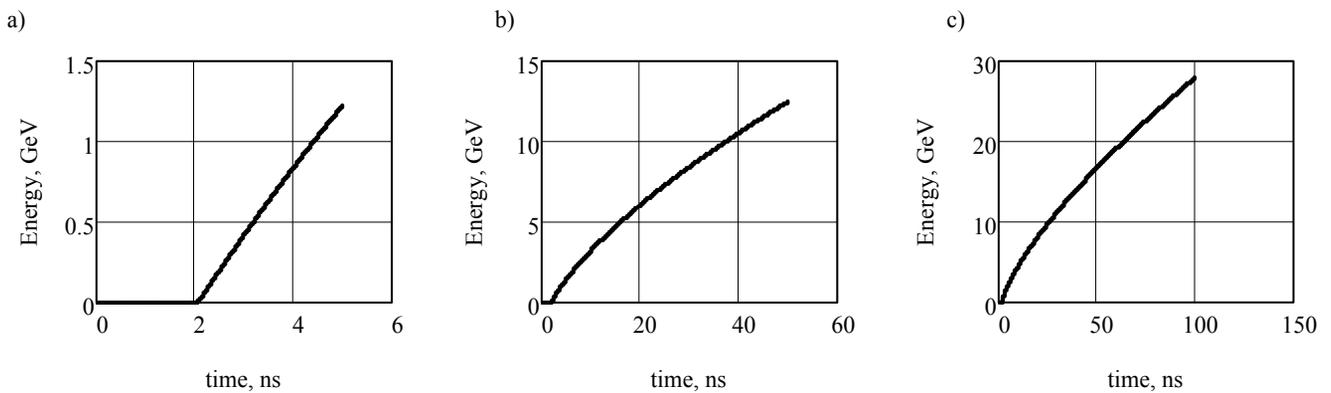


Fig. 5. Variation in time of accelerated proton kinetic energy gained as a result of interaction with laser beam of wavelength  $\lambda = 1 \mu\text{m}$  (a),  $\lambda = 10 \mu\text{m}$  (b), and for maser  $\lambda = 1 \text{mm}$  (c). Static axial magnetic field  $B_z = -300$  T (a) and  $-200$  T (b, c). Amplitude of electric field of electromagnetic wave  $E_0 = 10^{10}$  V/m

We also present some examples of kinetic energy gained by the particles during the acceleration process. The graphical pictures of the changes in the energy during the interaction of the mentioned fields are depicted in Fig. 4 for an electron and in Fig. 5 for a proton. For convenience the energy and time are presented in absolute (SI) units.

As it can be expected (Fig. 5), the noticeable level of a proton energy starts to increase far later than it is in the case of the electron (Fig. 4). However, it takes a relatively long time to gain a significant kinetic energy by both accelerated particles. First of all, for a proton but as well as for an electron one should expect the acceleration should last tens of nanoseconds in order to achieve energies of GeV level. Inertia of the accelerated particles plays a significant role. In order to shrink the acceleration time the amplitude  $E_0$  of the laser radiation should be raised.

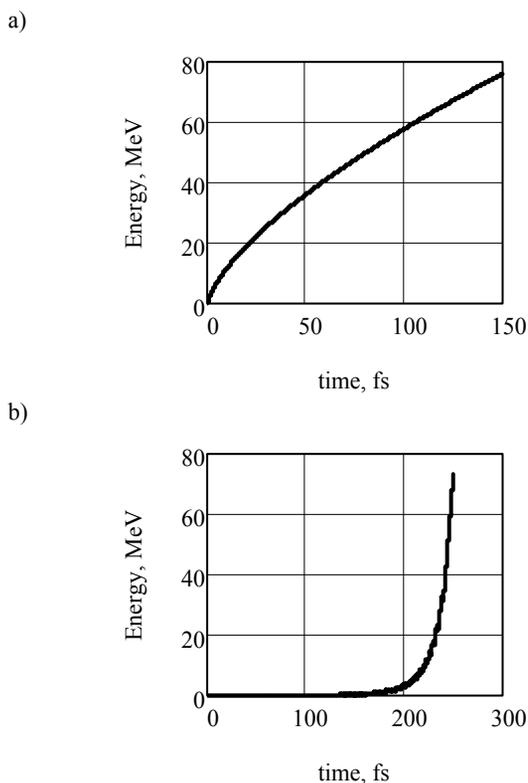


Fig. 6. Variation in time of accelerated electron (a) and proton (b) kinetic energy gained as a result of interaction with laser beam of wavelength  $\lambda = 760$  nm, and static axial magnetic field  $B_z = 50$  T (a) and  $-50$  T (b). Amplitude of electric field of electromagnetic wave  $E_0 = 10^{13}$  V/m (a) and  $E_0 = 5 \times 10^{13}$  V/m (b)

Finally we present the behaviour of electron (Fig. 6a) and proton (Fig. 6b) under action of already existing lasers or rather near to them. From Fig. 6 we can see that due to the rather short time of acceleration process (pulsing action of laser) we cannot expect for electrons or protons to gain very large energy. To obtain higher energies the particles should be accelerated longer or the amplitude  $E_0$  should be higher.

## 4. Discussion

Eqns (4) completely describe components of position and velocity of the accelerated particle as well as its total velocity under the interaction of the laser and static magnetic fields. Electromagnetic field of amplitudes ranging from  $E_0 = 10^9$  to  $E_0 = 5 \times 10^{13}$  V/m have been used in calculations. The presented above figures show the variation in time of these parameters. General remarks result from the difference of masses of considered particles. For proton it takes longer time to achieve a defined velocity than for electron. Due to the Doppler Effect, Fig. 1 showing the laser frequency 'seen' by the particle, indicates the smaller changes in frequency for a proton than for an electron. Longer time is required for proton to surmount the same distance than for electron (Fig. 3).

The energy gained by the charged particle due to the interaction with the laser or maser beam and the static magnetic field is calculated using the relativistic formula (6). When the total reduced velocity  $\beta$  of the electron or proton approaches the value nearly one, the gained kinetic energy (Eq. (6)) rapidly raises. There are the initial periods of the acceleration process shown, if the process is going on, larger energy gains are expected. We should be aware of including the Doppler effect in calculations, since neglecting it results in different values of kinetic energy gained by a particle, especially near or within the relativistic region. It was found impossible to perform comparison between the results of calculation with and without the Doppler Effect since neglecting the Doppler Effect leads to the energy oscillations strongly dependent on  $B_z$  and its direction [14, 15]. The comparison is rootless because the Doppler Effect exists in the real experiments and it was found to be especially significant in the relativistic region. However, some papers have been published with neglecting this effect [14-17]. The reason for doing this is the possibility to derive analytical solutions of the acceleration problem. This enables understanding of the basics of the numerous parameters impact on the particle acceleration.

The energy gained by a particle increases with the acceleration time. We should notice that during circulation the particle in its helical motion should not go outside the laser beam. This can be achieved applying the above-mentioned axially directed static magnetic field. The larger the field intensity is the smaller circulation radius of the particle is. In some cases the magnetic field intensity should be extremely high in order to keep a particle inside the laser or maser beam during the acceleration process. Help can be expected in the fact that at the boundary region of the beam, the amplitude  $E_0$  can be expected to be declining, this would result in the smaller radius of the trajectory that means the particle would go back to the beam. Thus, the magnetic field of reduced intensity may be sufficient.

Next remarks concerning  $B_z$  are connected with its direction. Provided it is axial, it was found that  $B_z > 0$  gave higher kinetic energy than  $B_z < 0$  for electrons and  $B_z$  direction had no noticeable impact on the proton energy. The amplitude of electric field  $E_0$  has an impact on duration that the charged particle gains the energy, the larger amplitude is, the less time is required to obtain the defined level of the kinetic energy.

The charged particles acceleration process is closely connected to the distance which the particle has to travel in order to get a desired kinetic energy. Since an electron velocity

component  $V_z$  almost through the whole acceleration process is close to the light velocity  $c$  the distance  $z(t)$  increases almost independently on the magnetic field or laser intensity. Approximately, a simple product  $ct$  is equal to  $z(t)$  for the electron. It is not the case for the proton, since its velocity remains rather far away from the light velocity through the significant part of the acceleration time.

## References

- [1] B.A. Remington, D. Arnett, R.P. Drake, H. Takabe, Modelling astrophysical phenomena in the laboratory with intense lasers, *Science* 284 (1999) 1488-1493.
- [2] P. Baum, A.H. Zewail, Attosecond electron pulses for 4d diffraction and microscopy, *Proceedings of the National Academy of Sciences of the United States of America - PNAS* 104 (2007) 18409-18414.
- [3] J.D. Lindl, P. Amendt, R. Berger, G. Glendinning, S.H. Glenzer, S.W. Haan, R. Kauffman, O.L. Landen, L.J. Suter, The physics basis for ignition using indirect-drive targets on the national ignition facility, *Physics of Plasmas* 11 (2004) 339-349.
- [4] K.W.D. Ledingham, P. McKenna, R.P. Singhal, Applications for nuclear phenomena generated by ultra-intense lasers, *Science* 300 (2003) 1107-1111.
- [5] F.V. Hartemann, S.N. Fochs, G.P. Le Sage, N.C. Luhmann, Jr., J.G. Woodworth, M.D. Perry, Y.J. Chen, A.K. Kerman, Nonlinear ponderomotive scattering of relativistic electrons by an intense laser field at focus, *Physical Review E* 51 (1995) 4833-4843.
- [6] J.J. Xu, Q. Kong, Z. Chen, P.X. Wang, D. Lin, Y.K. Ho, Vacuum laser acceleration in circularly polarized fields, *Journal of Physics D: Applied Physics* 40 (2007) 2464-2471.
- [7] K.P. Singh, Electron acceleration by an intense short pulse laser in a static magnetic field in vacuum, *Physical Review E* 69 (2004) 056410-1-056410-5.
- [8] Y.I. Salamin, Single-electron dynamics in a tightly focused laser beat wave: acceleration in vacuum, *Journal of Physics B: Atomic, Molecular and Optical Physics* 38 (2005) 4095-4110.
- [9] J. Fan, W. Luo, E. Fourkal, T. Lin, J. Li, I. Veltchev, C.-M. Ma, Shielding design for a laser-accelerated proton therapy system, *Physics in Medicine and Biology* 52 (2007) 3913-3930.
- [10] M. Borghesi, J. Fuchs, O. Willi, Laser-accelerated high-energy ions: state-of-the-art and applications, *Journal of Physics: Conference Series* 58 (2007) 74-80.
- [11] Y.I. Salamin, Z. Harman, C.H. Keitel, Direct high-power laser acceleration of ions for medical applications, *Physical Review Letters* 100 (2008) 155004-155008.
- [12] D.N. Gupta, H.J. Jang, H. Suk, Combined effect of tight-focusing and frequency-chirping on laser acceleration of an electron in vacuum, *Journal of Applied Physics* 105 (2009) 106110-1-106110-3.
- [13] J.R. Harris, D. Blackfield, G.J. Caporaso, Y.-J. Chen, S. Hawkins, M. Kendig, B. Poole, D.M. Sanders, M. Krogh, J.E. Managan, Vacuum insulator development for the dielectric wall accelerator, *Journal of Applied Physics* 104 (2008) 023517-023517-4.
- [14] A. Dubik, M.J. Małachowski, Basic features of a charged particle dynamics in a laser beam with static axial magnetic field, *Opto-Electronics Review* 17/4 (2009) 275-286.
- [15] A. Dubik, M.J. Małachowski, Resonance acceleration of a charged particle in a laser beam and static magnetic field, *Journal of Technical Physics* 50/2 (2009) 75-98.
- [16] Y.I. Salamin, F.H.M. Faisal, Ch.H. Keitel, Exact analysis of ultrahigh laser-induced acceleration of electrons by cyclotron autoresonance, *Physical Review A* 62 (2000) 053809-15.
- [17] A. Dubik, Movement of charge particles in electromagnetic field, Monograph No 101, Published at Radom University of Technology, Radom, 2007.