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# The influence of fill factor on the phononic crystal eigenfrequencies

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## ABSTRACT

**Purpose:** The aim of this article is to determine the effect of basic cell fill factor change on the eigenfrequencies observed in two-dimensional phononic crystal.

**Design/methodology/approach:** To perform simulation, the FDFD (finite difference frequency domain) algorithm was used. On this basis, the search for eigenfrequencies was carried out starting from lowest possible acoustic frequency range (~20 Hz) and limited to first nine search results found (up to nearly 2.2 kHz) for increasing fill factor while maintaining the shape of a rod inside cell.

**Findings:** The fill factor has a significant influence on the eigenfrequencies of the studied system when the frequency is above 1 kHz. With the increase of this factor at relatively low frequencies (less than 1 kHz in this case) there were no major changes observed.

**Research limitations/implications:** The results were found only for specific system consisting of materials with similar sound velocity. Therefore, more research should be carried out for other cases i.e. taking into account the different topology of primary cells and various materials with other propagation velocity of acoustic waves in these mediums.

**Practical implications:** Simulation of two-dimensional phononic crystal systems allows for designing new specialized multi-component materials with various acoustic properties. These systems can be adapted in a variety of applications, including acoustic filters, slow-wave devices, acoustic autocollimators and many other.

**Originality/value:** Basic research allow to improve the quality of knowledge on more advanced problems. For this reason, it is important to know in detail how simple systems work and to determine the basic properties of these systems.

**Keywords:** CAD/CAM; Computational Material Science; Phononic crystal; Eigenfrequencies

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

## 1. Introduction

Phononic crystals (PC) are structures that have interesting properties related to the interaction with the acoustic wave. These structures as well as their crystalline equivalents are characterized by translationally invariant translation vector of the lattice and local periodic distribution of properties associated with propagation of the acoustic wave, such as material density or velocity of sound. Materials of this type are extensively studied because of its wide range of possible applications including, inter alia waveguides [1-4], sensors [5-10], devices utilizing surface acoustic waves [11-13], acoustic collimators [14] and the filters of selected frequencies due to the presence of acoustic band gap [15-19]. The acoustic wave transmission in phononic crystals is influenced not only by their topological construction which is very important, but also by the material from which the individual elements and its surrounding medium are created [20,21]. This is due to the fact that the velocity of sound in a given medium is strongly dependent on its density. Not without significance is also the relationship between the speed of sound in the surrounding matrix to the other PC construction elements.

One of the key parameters of phononic crystal lattice cell is the ratio between the surface area of periodic element to the overall cell area called fill factor. This factor has a major influence on the construction of the basic lattice cell and therefore should also have a significant impact on the overall phononic crystal response to the acoustic waves. The vital role in this process have the eigenfrequencies of the basic unit cell. By the determination of the eigenfrequencies there is possibility to find out the band structure of the phononic crystal and thus other important related properties. Through the good knowledge of band gaps the variety of possible applications with great importance can be established. For example, a band structure (particularly acoustic band gaps) plays a huge role in the suppression of environment noise associated to a specific frequency like in the case of transformer devices or fast rotating machine parts.

One of the major groups of phononic crystals are those consisting of the combination of fluid and solid phases. The main difference between this group and the group consisting of phononic crystals based on the combination of the gas and the solid phases is generally smaller difference between the densities of its components. This entails obvious consequences on the properties of dispersion characteristics similar as in the case of photonic crystals, where there is a difference in the optical densities (and refractive indexes). In the fluid/solid system, where the solid element is embedded in fluid matrix, the coupled longitudal and transverse vector waves propagate in the solid elements.

Recently, much attention has been paid to the eigenfrequency optimization problems due to possibility of band gap enhancement, that is to make the widest possible separation of transmission bands in the crystal lattice. This is directly related with the distance on the frequency scale between successive eigenfrequencies of the studied system. Of course, as mentioned earlier, because the eigenfrequencies of the lattice are affected by the cell (and matrix) material, topological arrangement, shape and the size of component elements it is important to explore all the possibilities of influencing acoustic properties of the latter and thereby on the whole phononic crystal. With proper design, PC's can be used in many useful specialized applications.

In this paper the influence of fill factor on the eigenfrequencies of two material component phononic crystal is studied. Results are presented for eigenvalue problems of two-dimensional regular lattice PC using finite element method in the frequency range up to approximately 2.2 kHz.

## 2. Method

Based on the Bloch's theorem, the acoustic wave propagating in the periodic structure of phononic crystal must satisfy the periodic conditions on the edges of the cell (boundary conditions):

$$\psi(r) = e^{ik \cdot r} u(r) \tag{1}$$

where:

$\Psi$ - is the Bloch wave;

$r$ - is the position in the cell;

$u$ - is periodic function in accordance with periodicity of the lattice unit cell;

$k$ - is the crystal wave vector;

$e$ - is the Euler's number.

The eigenfrequency study is based upon following equation:

$$\nabla \cdot \left( -\frac{1}{\rho_c} \nabla p \right) + \frac{\lambda^2 p}{\rho_c c^2} = 0 \tag{2}$$

where:

$\lambda$ - is the eigenvalue related to the eigenfrequency $f$;

$\rho_c$ - is the density of medium;

$p$ - is the pressure field of sound wave;

$c$ - is the velocity of sound.

The first step taken to define the fill factor relationship to the eigenfrequencies was to prepare the primary lattice cell. Figure 1 shows the primary cell used in simulation.

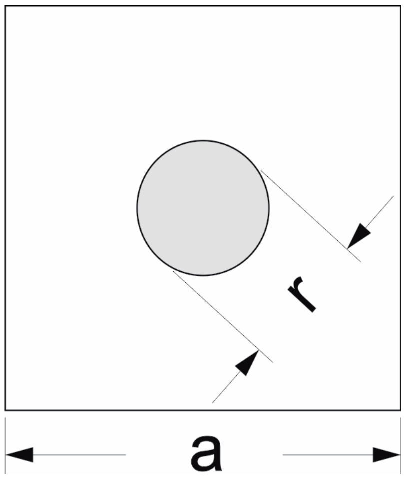


Fig. 1. Phononic crystal primary lattice cell. The rod inside the cell is placed centrally

The *a* value is the size of the unit cell in both directions (rectangular lattice) and the *r* is the diameter of circular rod placed centrally inside the cell. According to this, the fill factor α will be defined as:

$$\alpha = \pi\left(\frac{r}{2}\right)^2 / a^2 \qquad (3)$$

Each of the cell boundaries satisfies the Bloch periodic conditions.

As the medium material the water was used and as the rod material we have used mercury. Table 1 shows major properties related to the materials used in simulation.

Table 1.
Some key properties of used mediums / materials

| Medium | Speed of sound, m/s | Density, kg/m$^3$ |
|---|---|---|
| water | 1500 | 1000 |
| mercury | 1450 | 1350 |

As can be seen in Table 1 chosen materials exhibit similar properties (related to simulation).

The simulation was performed for the fill factors given in Table 2.

Table 2.
Fill factors and rod radius (in the units of cell size *a*)

| Number | Rod radius, a | Fill factor, α |
|---|---|---|
| 1. | 0.08 | 0.022 |
| 2. | 0.20 | 0.126 |
| 3. | 0.25 | 0.196 |
| 4. | 0.30 | 0.283 |
| 5. | 0.35 | 0.385 |
| 6. | 0.40 | 0.503 |
| 7. | 0.45 | 0.636 |

## 3. Simulation setup

The simulation was performed using finite difference method in frequency domain. The simulation space was covered by a ultrafine mesh consisting of 29652 triangular elements. The simulations were performed for the ambient temperature of 293.15 K with the initial reference pressure of 1 atm. The eigenfrequency search was limited to the frequency range from 20 Hz up to 2.2 kHz or to first nine eigenfrequencies. Size of primitive cell *a* = 1 m.

## 4. Results

Figures 2 to 8 show the simulation results (omitting repeated mods for the same frequency) for the single cell of phononic crystal with a variable *α* parameter.

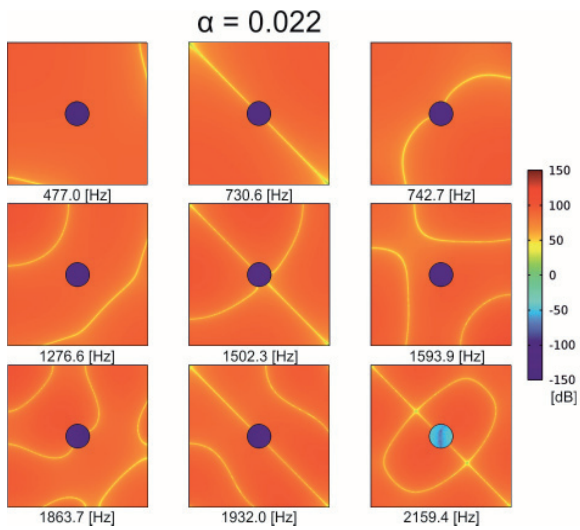


Fig. 2. First nine eigenfrequencies for fill factor α=0.022a

As can be seen in Fig. 2 in most cases only the aqueous matrix showed existence of resonant modes. The first encountered eigenfrequency from the audible range was for 477.0 Hz. In the last case of 2159.4 Hz a new mod can be observed inside the mercury rod with relatively low SPL (sound pressure level), defined as:

$$SPL = 10\log\frac{\langle p^2 \rangle}{p_0^2} \tag{4}$$

where:

$\langle p^2 \rangle$ – is the mean square of sound pressure;

$p_0 = 2 \cdot 10^{-5} [Pa]$ – is the reference pressure value.

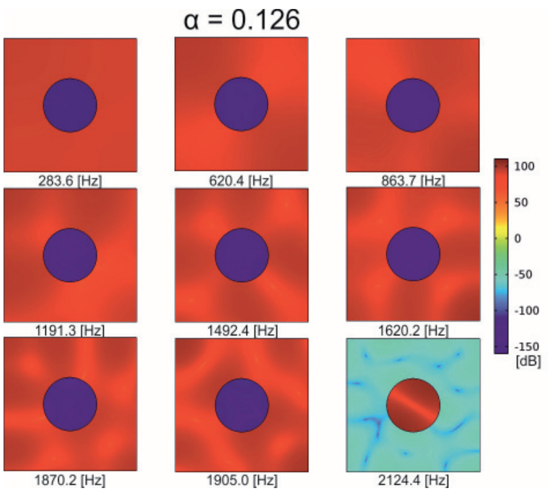


Fig. 3. First nine eigenfrequencies for fill factor α=0.126a

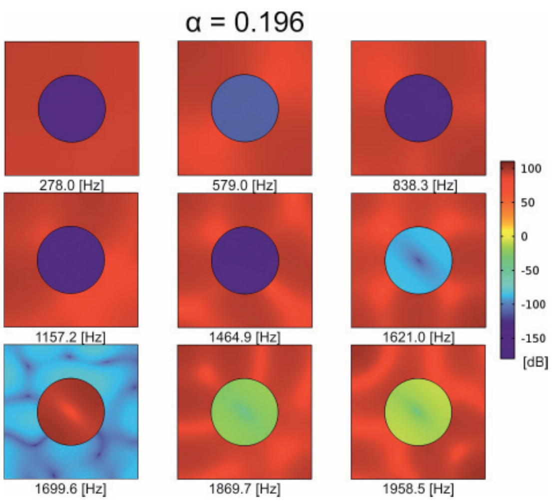


Fig. 4. First nine eigenfrequencies for fill factor α=0.196a

Increase in fill factor from 0.022 to 0.126 caused strong blurring of eigenvalues pattern. The difference between maximum and minimum values of SPL in this case decreases. For the highest observed frequency (2124.4 Hz) the new mod inside the rod emerges, while simultaneously a drop of SPL in surrounding matrix can be seen.

Figure 4 reveals further deepening of the blurring degree related to the aqueous matrix with an increase in the size of mercury rod. A repetitive pattern associated with standing wave inside rod can be seen starting from 1621 Hz eigenfrequency. Similar pattern also appeared in Fig. 3 for the highest frequency. For the first five eigenstates (up to 1464.9 Hz) there is an complete uniform distribution of SPL pressure in this rod. Such behavior of pressure level is clearly connected with high level of symmetry.

Witch further increase of fill factor once again a sharp image of standing wave patterns can be seen (Fig. 5). Less even distribution of pressure inside PC cell while maintaining high pattern symmetry indicates, that in the fill factor - pattern blurring relationship there are many local minima that are occurring periodically.

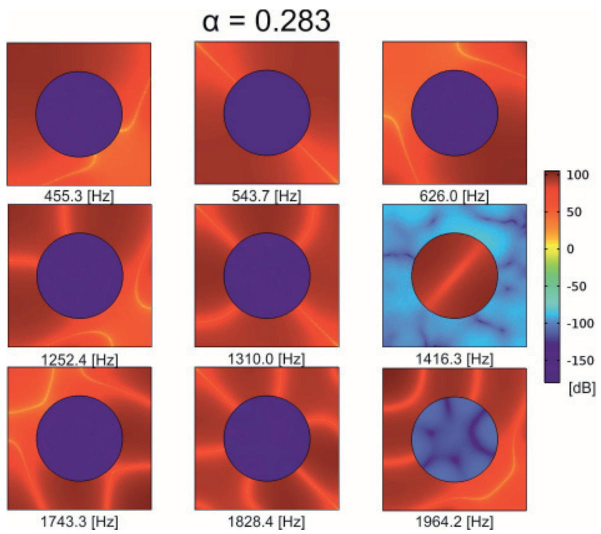


Fig. 5. First nine eigenfrequencies for fill factor α=0.283a

Figure 6 shows that while still increasing fill factor, more high symmetry patterns of standing waves inside the rod are emerging. Additionally, as expected aqueous matrix patterns are more blurred.

The patterns from Fig. 7 and Fig. 8 not differ significantly from previously discussed cases. In general, the impact of the fill factor is quite complicated and shows a certain repeatability. The increase in fill factor causes emergence of high symmetry patterns both in the environment and inside the mercury rod. Also there can be seen a repetitive alternating effect of blurring and sharpening of the phononic crystal cell matrix patterns, which means that the differences in sound pressure level are alternating to be respectively smaller and larger.

In Table 3 the eigenfrequency data with the corresponding fill factors were collected.

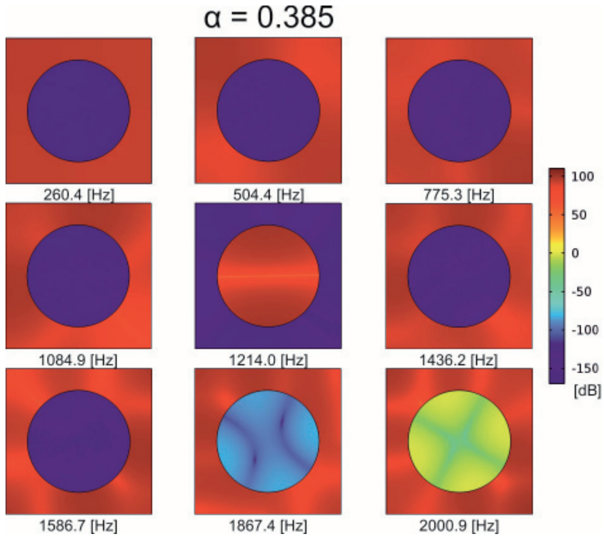


Fig. 6. First nine eigenfrequencies for fill factor α=0.385a

From analysis of data collected in Table 3, the lowest observed eigenfrequency was for the fill factor α= 0.385 and the highest for α= 0.022. Figure 9 shows first eigenfrequency in function of fill $q$ factor (data from Table 3).

As can be seen in Fig. 9 the relationship between eigenfrequency in function of fill factor for first eigenfrequency (starting from 477.0 Hz) is rather complicated. Looking more closely a dependence manifested by decrease in overall frequencies is visible. This "zigzag" like trend which is nearly linear is fairly difficult to explain, but is definitely associated with blurring effect visible on Figs. 3,4,6 and 8. When the blurring effect occurs the eigenfrequencies presented on Fig. 9 are shifted to the low frequency region. It should be remembered that phononic crystal is made from many of the primary cells packed side by side in every direction of two-dimensional space. In that case a possible explanation for frequency shifting this is that when fill factor is rising the acoustic standing waves are changing their frequencies allowing for the formation of the standing waves only at the specific frequencies (when due to the wavelength nodes are formed on the surface of rods).

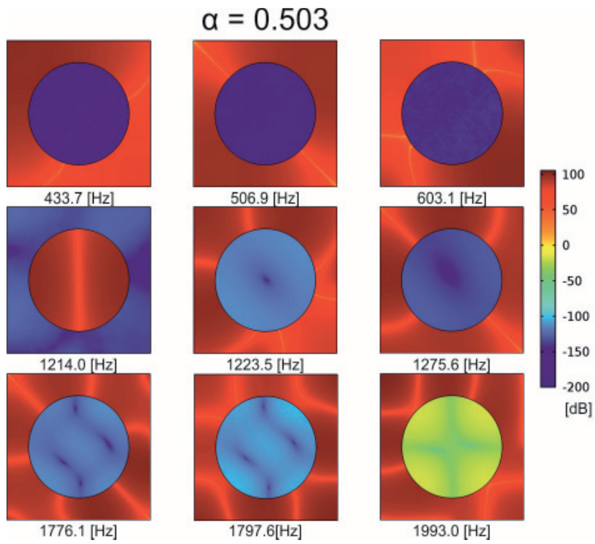


Fig. 7. First nine eigenfrequencies for fill factor α=0.503a

Table 3.
Eigenfrequency values for considered fill factors

| Fill factor, a | Freq., Hz 1 | Fill factor, a | Freq., Hz 2 | Fill factor, a | Freq., Hz 3 | Fill factor, a | Freq., Hz 4 | Fill factor, a | Freq., Hz 5 | Fill factor, a | Freq., Hz 6 | Fill factor, a | Freq., Hz 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.022 | 477.0 | 0.126 | 283.6 | 0.196 | 278.0 | 0.283 | 455.3 | 0.385 | 260.4 | 0.503 | 433.7 | 0.636 | 246.9 |
| | 730.6 | | 620.4 | | 579.0 | | 543.7 | | 504.4 | | 506.9 | | 465.9 |
| | 742.7 | | 863.7 | | 838.3 | | 626.0 | | 755.3 | | 604.1 | | 722.1 |
| | 1276.6 | | 1191.3 | | 1157.2 | | 1252.4 | | 1084.9 | | 1214.0 | | 1061.2 |
| | 1502.3 | | 1492.4 | | 1464.9 | | 1310.0 | | 1214.0 | | 1223.3 | | 1062.2 |
| | 1593.9 | | 1620.2 | | 1621.0 | | 1416.3 | | 1436.2 | | 1275.6 | | 1415.6 |
| | 1863.7 | | 1870.2 | | 1699.6 | | 1743.3 | | 1586.7 | | 1776.1 | | 1586.1 |
| | 1932.0 | | 1905.0 | | 1869.7 | | 1828.4 | | 1867.4 | | 1797.6 | | 1762.1 |
| | 2159.4 | | 2124,4 | | 1958.5 | | 1964.2 | | 2000.9 | | 1993.0 | | 1762.1 |

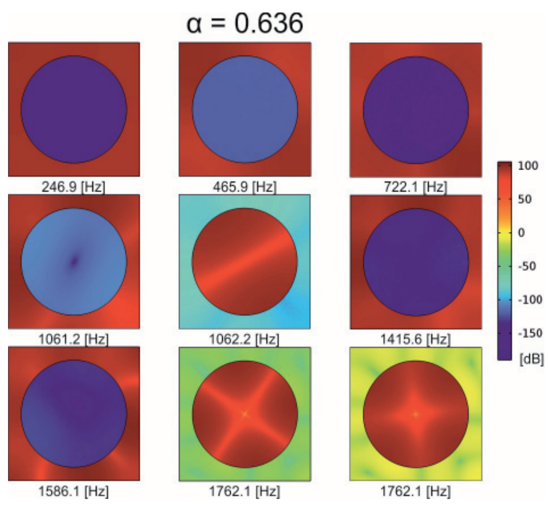


Fig. 8. First nine eigenfrequencies for fill factor α=0.636a

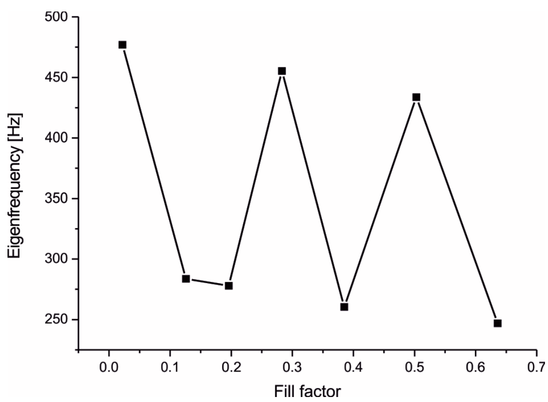


Fig. 9. Fill factor vs. eigenfrequency for first eigenfrequency. The presented lines are only guidelines for the eyes

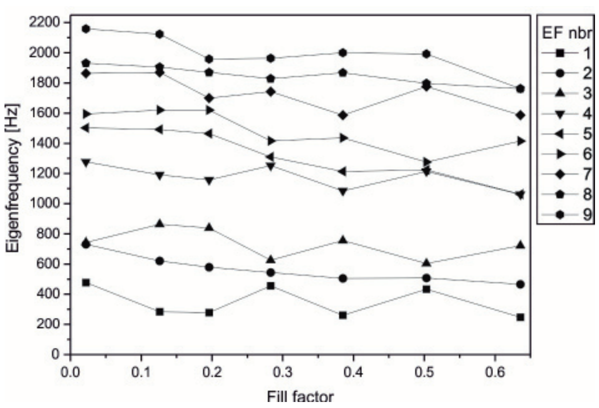


Fig. 10. Fill factor vs. eigenfrequency for first nine eigenfrequencies. The presented lines are only guidelines for the eyes

In order to verify whether the effect of frequency shifting occurs for other fill factors the Fig. 10 showing all eigenfrequencies was made.

Analysis of Fig. 10 shows that frequency shifting effect has different influence on all of the curves. In some cases (curve number 1 and 3) a anti symmetry relation is visible. Moreover in each case there is a decreasing in eigenfrequency value with the increase of fill factor.

## 5. Conclusions

In this paper the influence of fill factor on the phononic crystal eigenfrequencies was studied by using the FEM eigenfrequency method. The study identified that when the fill factor is increasing the eigenfrequencies tend in the direction of lower frequencies what is interesting and generally unexpected effect. It is normal that with decreasing space between the rods a standing waves of increasing frequencies should be formed. It should also be remembered that with increasing fill factor standing waves can also occur inside the rods. In that case, these two effects compete with each other, resulting in a decrease trend of the eigenfrequency.

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