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Particle shape influence on elastic-plastic behaviour of particle-reinforced composites

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ABSTRACT

Purpose: Particle-reinforced composite materials very often provide unique and versatile properties. Modelling and prediction of effective heterogeneous material behaviour is a complex problem. However it is possible to estimate an influence of microstructure properties on effective macro material properties. Mentioned multi-scale approach can lead to better understanding of particle-reinforced composite behaviour. The paper is focused on prediction of an influence of particle shape on effective elastic properties, yield stress and stress distribution in particle-reinforced metal matrix composites.

Design/methodology/approach: This research is based on usage of homogenization procedure connected with volume averaging of stress and strain values in RVE (Representative Volume Element). To create the RVE geometry Digimat-FE software is applied. Finite element method is applied to solve boundary value problem, in particular a commercial MSC.Marc software is used.

Findings: Cylindrical particles provide the highest stiffness and yield stress while the lowest values of stiffness and yield stress are connected with spherical particles. On the other hand stress distribution in spherical particles is more uniform than in cylindrical and prismatic ones, which are more prone to an occurrence of stress concentration.

Research limitations/implications: During this study simple, idealised geometries of the inclusions are considered, in particular sphere, prism and cylinder ones. Moreover, uniform size and uniform spatial distribution of the inclusions are taken into account. However in further work presented methodology can be applied to analysis of RVE that maps the real microstructure.

Practical implications: Presented methodology can deal with an analysis of composite material with any inclusion shape. Predicting an effective composite material properties by analysis of material properties at microstructure level leads to better understanding and control of particle-reinforced composite materials behaviour.

Originality/value: The paper in details presents in details an investigation of influence of inclusion shape on effective elastic-plastic material properties. In addition it describes the differences between stress distributions in composites with various inclusion shapes.

Keywords: Particle-reinforced composite; Shape effects; Homogenization; Micromechanics

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

Particle-reinforced metal matrix composite materials have found application in many areas of engineering practice. They usually provide higher strength, stiffness and weight savings in comparison with conventional metal alloys. Moreover this group of materials is attractive due to their cost-effectiveness and isotropic properties [1]. Estimation of elastic-plastic response in particle-reinforced composites is very complex issue because it depends on variety of factors such as particle size, shape, distribution or residual stresses. This study focuses on analysis of particle shape effects. The main objective of this work is an investigation of influence of particles shape on composite stiffness, yield stress and stress distribution in material phases. In order to estimate a stiffness of heterogeneous material, classical mean field methods can be used. This group of methods is based on well-known equivalent inclusion approach of Eshelby [1-3]. Advantage of this approach is computational efficiency and simplicity. On the other hand it has got limitations, for example it is generally restricted to analysis of inclusions of spheroid shape. Approach that can handle with analysis of any particle shape is finite [4-6] or boundary [8,9] element analysis of representative volume element (RVE). RVE is a statistical representation of material properties. It should contain enough information to describe behaviour of considered composite [8]. During this study a commercial software Digimat-FE is used to generate the RVE 3D geometry [10]. Created finite element models based on the geometry created in Digimat are solved by application of MSC.Marc software.

2. Analysis of the RVE

2.1. Geometry preparation

During this research three arbitrary shapes of inclusions, which geometries are presented in Fig. 1, are taken into account A geometry of the RVE is generated using Digimat-FE software [10]. Each RVE is a cube filled with randomly oriented 20 particles. Volume fraction of the particles is 0.2 and they are randomly distributed in space without interpenetration. Figs. 2-4 show distribution of particles in created RVEs.

2.2. Estimation of effective properties

To calculate the effective elasticity tensors of heterogeneous materials the usage of homogenization procedure is essential. The homogenization procedure involves replacing the heterogeneous material with an equivalent homogeneous material. Calculation of the equivalent material properties requires to solve six RVE boundary value problems (BVP) in three dimensional case. For each BVP a prescribed strain is applied in accordance with equation 1 (a superscript indicates the number of analysis).

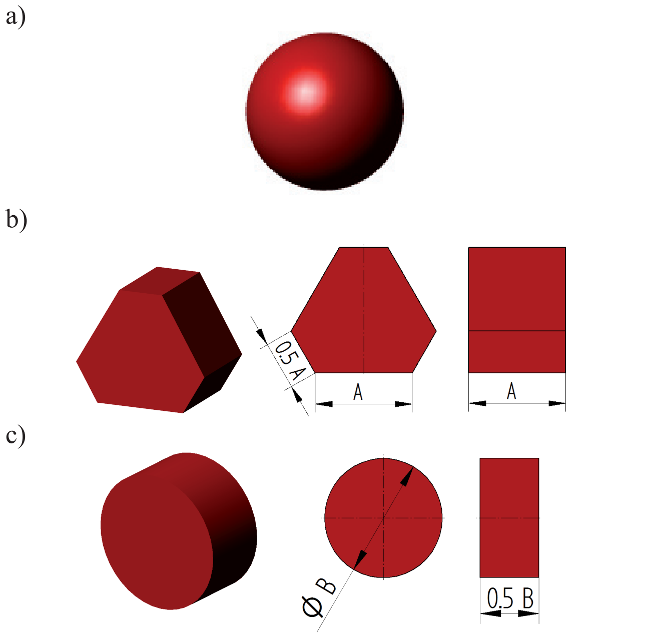

Fig. 1. Shapes of the inclusions: a) spherical; b) prismatic; c) cylindrical

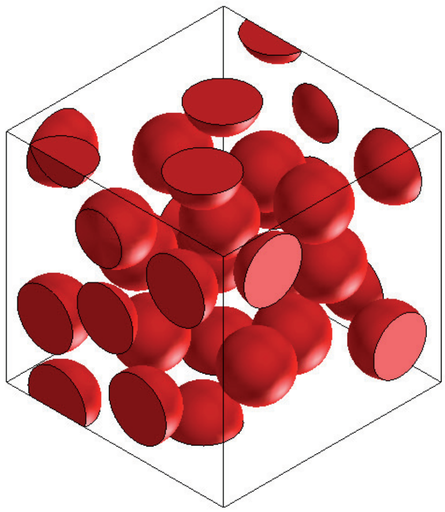

Fig. 2. RVE containing spherical particles

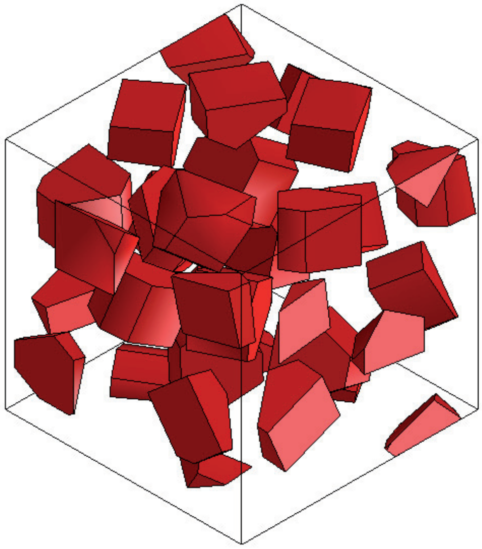

Fig. 3. RVE containing prismatic particles

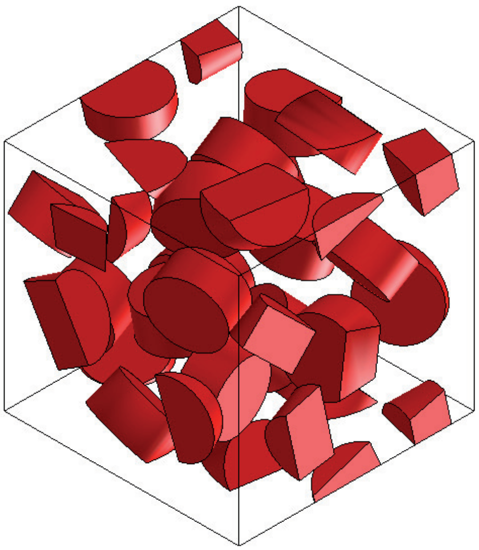

Fig. 4. RVE containing cylindrical particles

$$\varepsilon^{(1)}=\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}\ \varepsilon^{(2)}=\begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}\ \varepsilon^{(3)}=\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}\ \varepsilon^{(4)}=\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}\ \varepsilon^{(5)}=\begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}\ \varepsilon^{(6)}=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \tag{1}$$

In addition, periodic boundary conditions are introduced [2]. During this study, finite element method is applied to solve BVP, in particular commercial software MSC.Marc is used. Strains and periodic boundary conditions are prescribed via multi-point constraints (MPC). After solving six BVPs, both the stresses (2) and strains (3) are averaged in post-processing stage of the analysis [7].

$$<\sigma_{ij}>=\frac{1}{V_{RVE}}\int_{V_{RVE}}\sigma_{ij}dV_{RVE} \tag{2}$$

$$<\varepsilon_{ij}>=\frac{1}{V_{RVE}}\int_{V_{RVE}}\varepsilon_{ij}dV_{RVE} \tag{3}$$

where $<\sigma_{ij}>$ is average stress, $<\varepsilon_{ij}>$ is average strain, σ_{ij} is stress in the RVE, ε_{ij} is strain in the RVE and V_{RVE} is volume of the RVE.

Elasticity matrix C_{RAW} that binds average stress and average strains is expressed by:

$$<\sigma_{ij}>=C_{RAW}<\varepsilon_{ij}> \tag{4}$$

Due to the random orientation of the particles in space, isotropic elastic properties are expected. However, calculated tensor C_{RAW} is not perfectly isotropic and even symmetric. Symmetrisation of raw tensor is performed in accordance with:

$$C=\frac{C_{RAW}+C_{RAW}{}^{T}}{2} \tag{5}$$

To define isotropic material parameters approximate Lamé parameters are determined as isotropic approximation of an anisotropic stiffness tensor [11]:

$$\lambda=\frac{1}{15}\left[C_{11}+C_{22}+C_{33}-2(C_{44}+C_{55}+C_{66})+4(C_{12}+C_{13}+C_{23})\right] \tag{6}$$

$$\mu=\frac{1}{15}\left[C_{11}+C_{22}+C_{33}+3(C_{44}+C_{55}+C_{66})-(C_{12}+C_{13}+C_{23})\right] \tag{7}$$

The Young modulus and the Poisson ratio expressed by the Lamé parameters can be defined as follows [13]:

$$E=\frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \tag{8}$$

$$v=\frac{\lambda}{2(\lambda+\mu)} \tag{9}$$

To compare the results of finite element based homogenization with existing analytical models, a well known Mori-Tanaka mean field method is considered. Mori-Tanaka strain concentration tensor can be shown as [15]:

$$A^{MT} = S[(1-v_i)I + v_i S]^{-1} \tag{10}$$

and effective stiffness tensor of composite can be written in closed form as follows [9]

$$C = C_m + f_i[(C_i - C_m)A^{MT}(f_m I + f_i A^{MT})]^{-1} \tag{11}$$

where S is an Eshelby tensor, I is identity tensor, C_m and C_i are stiffness tensors of matrix and inclusion material, respectively, f_m and fi are volume fractions of matrix and inclusion phases. Eshelby tensor components for spherical inclusion can be written in explicit form as [16]:

$$S_{11} = S_{22} = S_{33} = \frac{7-5v_m}{15(1-v_m)} \tag{12}$$

$$S_{12} = S_{23} = S_{31} = S_{13} = S_{21} = S_{32} = \frac{5v_m - 1}{15(1-v_m)} \tag{13}$$

$$S_{44} = S_{55} = S_{66} = \frac{4-5v_m}{15(1-v_m)} \tag{14}$$

The remaining tensor components are zeros.

Investigation of inclusion shape influence on plastic behaviour of composites is conducted by enforcing uniaxial strain on the RVE. Three analyses, assuming different strains, are performed for each RVE:

$$\varepsilon_{11}{}^{(1)} = 0.015 \quad \varepsilon_{22}{}^{(2)} = 0.015 \quad \varepsilon_{33}{}^{(3)} = 0.015 \tag{15}$$

In this case the matrix constitutive behaviour is modelled as elastic-plastic and inclusion as linear elastic. Stress-strain curve is obtained as a result of each analysis. For each case 0.2% offset yield stress was determined. Finally, an effective 0.2% offset yield stress of composite is evaluated as arithmetic mean of that obtained in three analyses:

$$R_{p0.2}{}^{EFF} = \frac{1}{3}(R_{p0.2}{}^{(1)} + R_{p0.2}{}^{(2)} + R_{p0.2}{}^{(3)}) \tag{16}$$

2.3. Constituents properties and discrete model

Assumed properties of the composite constituents are presented in Tables 1 and 2. Both matrix and inclusion materials models are linear in analysis of effective elastic properties. Considered properties are typical for aluminium alloy 6061T6 and SiC particles. In case of estimating elastic-plastic response the matrix is modelled as elastic-plastic material and the inclusion as elastic material.

Fig. 5 shows finite element discretization of the RVE in case of analysis of spherical reinforcement. Approximately 83000 of tetrahedral finite elements with quadratic shape functions were created in all cases

Table 1.
Properties of the matrix material

Property	Value	Units
Modulus of elasticity	68.9	GPa
Poisson's ratio	0.35	
Yield stress	276	MPa
Hardening constant	255	MPa
Hardening exponent	0.3	

Table 2.
Properties of the inclusions material

Property	Value	Units
Modulus of elasticity	410	GPa
Poisson's ratio	0.19	

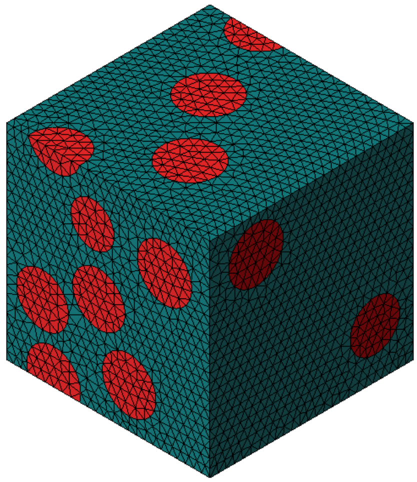

Fig. 5. Discretization of the RVE with spherical particles

3. Results and discussion

Computed normalised Young modulus and Poisson ratio of analysed representative volume elements are collected in Table 3. Normalised Young modulus and Poisson ratio are expressed as:

$$E = \frac{E_{eqv}}{E_m} \tag{17}$$

$$\nu = \frac{\nu_{eqv}}{\nu_m} \tag{18}$$

where subscripts *eqv* and *m* indicate homogenized equivalent properties and matrix properties, respectively.

Detailed analysis results in form of computed elastic tensors are shown for the case of the composite reinforced with spherical particles. Tensor $C^{(M\text{-}T)}$ (19) presents the results of Mori-Tanaka homogenization. Tensor $C_{RAW}{}^{(FE)}$ is the result of six finite element analysis of the RVE, the non-orthotropic terms are assumed implicitly as zeros. The best isotropic fit of obtained anisotropic $C_{RAW}{}^{(FE)}$ tensor is expressed in form of tensor $C_{ISO}{}^{(FE)}$ (21).

Table 3.
Material properties of analysed composite

Particle shape	Normalised Young modulus	Normalised Poisson ratio
Sphere (Mori-Tanaka)	1.347	0.940
Sphere (FEM)	1.371	0.933
Prism	1.417	0.922
Cylinder	1.462	0.867

$$C^{(M-T)} = \begin{bmatrix} 137208.89 & 67377.88 & 67377.88 & 0 & 0 & 0 \\ 67377.88 & 137208.89 & 67377.88 & 0 & 0 & 0 \\ 67377.88 & 67377.88 & 137208.89 & 0 & 0 & 0 \\ 0 & 0 & 0 & 34915.50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 34915.50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 34915.50 \end{bmatrix} \tag{19}$$

$$C_{RAW}{}^{(FE)} = \begin{bmatrix} 138156.02 & 66758.81 & 66659.03 & 0 & 0 & 0 \\ 66936.11 & 138269.70 & 66667.18 & 0 & 0 & 0 \\ 66773.79 & 66607.88 & 138222.77 & 0 & 0 & 0 \\ 0 & 0 & 0 & 35521.53 & 0 & 0 \\ 0 & 0 & 0 & 0 & 35386.65 & 0 \\ 0 & 0 & 0 & 0 & 0 & 35702.70 \end{bmatrix} \tag{20}$$

$$C_{ISO}{}^{(FE)} = \begin{bmatrix} 137989.97 & 66843.21 & 66843.21 & 0 & 0 & 0 \\ 66843.21 & 137989.97 & 66843.21 & 0 & 0 & 0 \\ 66843.21 & 66843.21 & 137989.97 & 0 & 0 & 0 \\ 0 & 0 & 0 & 35618.62 & 0 & 0 \\ 0 & 0 & 0 & 0 & 35618.62 & 0 \\ 0 & 0 & 0 & 0 & 0 & 35618.62 \end{bmatrix} \tag{21}$$

It can be observed that there is a difference between Mori-Tanaka and finite element based prediction of material stiffness tensors; finite element solution gives higher value of Young modulus. In comparison with spherical particles, prismatic and cylindrical reinforcement shapes provide higher material stiffness.

Assuming elastic-plastic behaviour of the matrix material, stress-strain curves of composite are shown in Fig. 6.

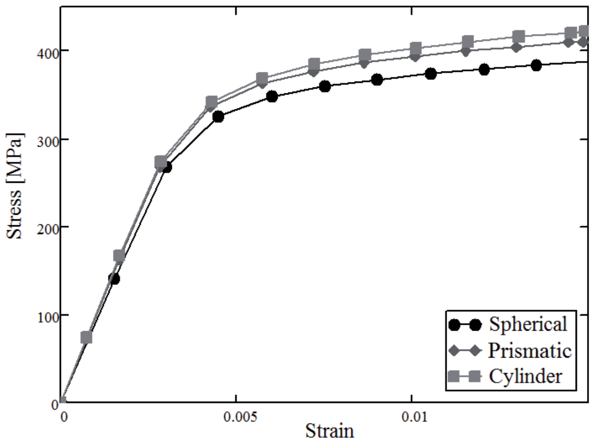

Fig. 6. Stress-strain curves for composites with different particle shapes, uniaxial tension.

Calculated equivalent 0.2% offset yield stresses for different reinforcement shapes are collected in Table 4. In addition, a standard deviation of data obtained in three uniaxial tension tests is presented. It can be observed that the yield stress of composite reinforced with prismatic and cylindrical particles is higher than yield stress of composite with spherical reinforcement.

Table 4.
Equivalent 0.2% offset yield stress for different reinforcement shapes, MPa

Particle shape	Yield stress, MPa	Standard deviation
Sphere	343.53	1.06
Prism	361.40	3.07
Cylinder	363.19	3.41

Fig. 7 shows maximum principal stress distribution in analysed reinforcements in case of uniaxial loading with prescribed 0.01 strain.

Fig. 8 and 9 show probability density distributions of stresses in the matrix and particle phases in case of uniaxial loading with prescribed 0.01 strain. Analysis of Fig. 8 leads to the conclusion that there are no significant differences between stress distributions in the matrix computed for analysed reinforcement shapes. However comparison of stress distribution in the particle phase, shown in Fig. 9, suggests that stress distribution in spherical particles is significantly different than in prismatic and cylindrical ones. The probability of occurrence of high stress values is higher for prismatic and cylindrical particles than for spherical particles which provide more uniform character of stress distribution.

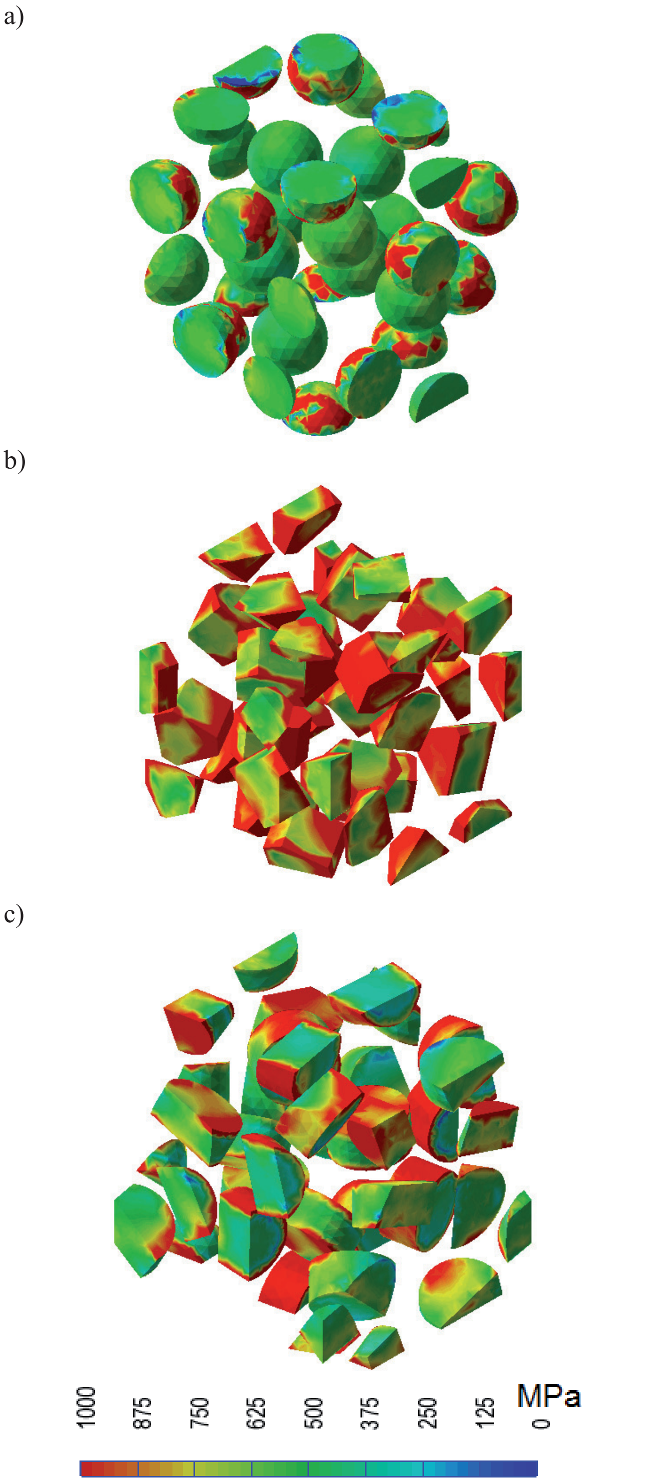

Fig. 7. Maximum principal stress distribution in a) spherical, b) prismatic, c) cylindrical particles (uniaxial loading with prescribed 0.01 strain, maximum visualised value is 1 GPa)

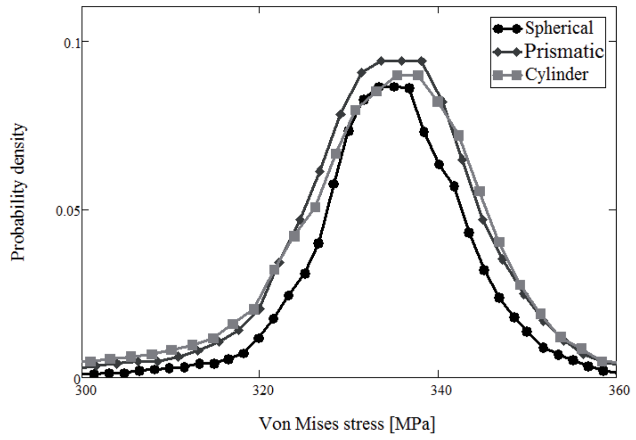

Fig. 8. Probability density distribution of Von Mises stresses in the matrix (uniaxial loading with prescribed 0.01 strain)

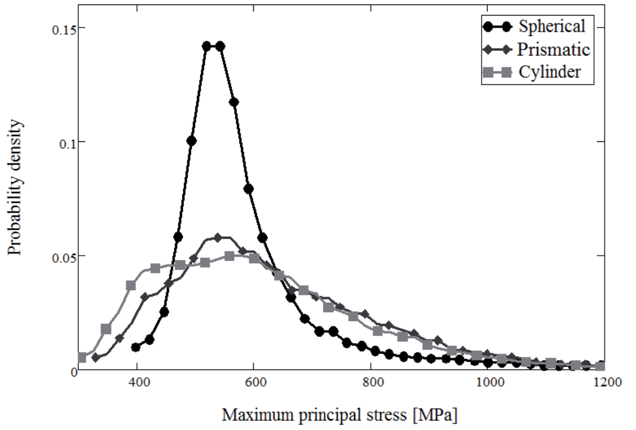

Fig. 9. Probability density distribution of maximum principal stresses in the particles (uniaxial loading with prescribed 0.01 strain)

4. Conclusions

During this study the elastic-plastic effective properties of composites reinforced with particles of different shape were investigated. Conducted finite element analysis of the RVE allowed to take into account an arbitrary shape of the particle. Three different particle shapes were considered: spherical, prismatic and cylindrical. Cylindrical particles provide the highest stiffness and yield stress while the lowest values of stiffness and yield stress are connected with spherical particles. On the other hand, the stress distribution in spherical particles is more uniform than in cylindrical and prismatic ones. In other words in cylindrical and prismatic particles stress locally reaches higher values than in spherical particles. Predicting an effective composite material properties by analysis of material properties at microstructure level can lead to better understanding and control of particle-reinforced composite materials behaviour.

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