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# Influence of rod diameter on acoustic band gaps in 2D phononic crystal

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## ABSTRACT

**Purpose:** The purpose of this paper is to investigate influence of changing the fill factor (or rod diameter) on acoustic properties of phononic crystal made of mercury rods inside of water matrix. Change in construction of primary cell without changing the shape of rod may cause shifts in bands leading to widening of forbidden band gaps, which is the basis of modern composite material designing process.

**Design/methodology/approach:** Band structure is determined by using the finite element study known as finite difference frequency domain simulation method. This is achieved by virtual construction and simulation of primary cell of phononic crystal. Phononic crystals are special devices which by periodic arrangement of properties related to the sound can affect the transmission of acoustic waves thru their body.

**Findings:** The fill factor/rod diameter has a significant influence on the acoustic band structure of studied phononic crystal which can be divided in two mainly effects: fission and compression of band structure.

**Research limitations/implications:** In order to better understand basic properties of phononic crystals and to get full control over the band gaps a series of similar calculations should be done for broader range of frequencies covering both infrasound and ultrasound wavelength regions. Also structures of other cut shape of rod and different primary cell structure resulting in diverse phononic crystal structure should be investigated in the future.

**Practical implications:** Phononic crystals are important devices in variety of applications ranging from noise control through acoustic computing, health applications and entertainment up to military applications. Therefore full knowledge about specific working conditions and elementary properties is necessary for complete control in targeted applications. Controlling the fill factor is one of the simplest methods to achieve specific band gap positions and widths.

**Originality/value:** The novelty is in use of different phase materials with similar acoustic characteristics affecting the whole sonic properties of device manifested by their calculated band structure. The target group are scientists interested in practical applications of various acoustic materials.

**Keywords:** CAD/CAM; Computational material science; FDFD; Acoustic band structure; Phononic crystals

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## PROPERTIES

## 1. Introduction

Phononic crystals are recently intensively investigated due to their numerous application possibilities [1-6]. The first experimental confirmation of their interesting properties was in 1979 when Narayanamurti et al. studied phonons of high frequency through GaAs/AlGaAs supperlattice [7]. Since then, much more research is carried out by simulation [8,9] which can provide reliable data in a much shorter period of time. Among the many methods for simulation of acoustic devices, some basic methods can be distinguished, such as: geometrical tracing, finite difference in time and frequency domain and fully analytical approach. Each has its advantages and disadvantages but most of them are considered twofold reasons: computational cost and calculation precision. The most commonly used methods are those based on finite elements as they maintain a balance between speed and accuracy.

Phononic crystals are used widely in variety of applications such as focusing devices, ultrasound acoustic imaging, directional speakers, noise shielding via dispersion or band gaps, focusing devices and acoustic lenses, waveguides and many more [10-14].

To create a phononic crystal a diversity in spatial positions of repetitive elements distributed periodically as it is in other known crystal structures is needed. This can include pseudo one-dimensional multilayer systems (supperlattices), 2D periodic lattices and three dimensional crystal structures. A three types of systems can be distinguished: liquid/liquid, liquid/solid and solid/gas phases compositions. All of them depending on their construction are characterized by different properties and therefore are used in different environments. However, regardless of the use they must have a common features, in particular their constituent components must vary in acoustic properties.

## 2. Governing equations

Finite Difference Method allows to simulate the propagation of acoustic waves in almost every medium and is widely used in many studies related to acoustic simulations [24,25].

To describe the behavior of acoustic waves in a particular environment, it is necessary to start from the first order differential equations [10]:

$$\kappa \frac{\partial}{\partial t} p(\hat{x},t) = \nabla \cdot \hat{u} \tag{1}$$

$$\rho_0 \rho_r \frac{d}{dt} \hat{u}(\hat{x},t) = \nabla p(\hat{x},t) \tag{2}$$

where:
$p(\hat{x},t)$ – pressure field $[F/m^2] = [kg/(m-\sec^2)]$,
$\hat{u}(\hat{x},t)$ – is the wave velocity vector [ $m/s$ ],
$\rho_0$ – density of medium,
$\rho_r$ – is the density relative to $\rho_0$,
$\kappa$ – is the compressibility $\kappa = 1/(\rho_0 \cdot \rho_r \cdot c^2)$,

Moving on to the frequency domain pressure $p$ in the position $r$ is given by the equation:

$$p(r) = p(x,y)e^{ik,z} \tag{3}$$

where:
$r$ – is the position vector of spatial coordinates $(x,y)$ respectively, for the x and y-axis,
$k_z$ – is the wave number perpendicular to the plane.

According to equation (3), the general case equation is given as:

$$\nabla \cdot \left( -\frac{1}{\rho_c} \left( \nabla_p - q_d \right) \right) - \frac{\omega^2 p}{\rho_c c_c^2} = Q_m \tag{4}$$

where:
$p$ – is the pressure,
$\rho_c$ – density of medium,
$c_c$ – speed of sound,
$q_d$ – dipole sound source (in this case = 0),
$Q_m$ – monopole sound source.

For the two-dimensional simulation equation (4) can be written in the following form:

$$\nabla \cdot \left( -\frac{1}{\rho_c} \left( \nabla_p - q_d \right) \right) - \frac{1}{\rho_c} \left( \frac{\omega^2}{c_c^2} - k_z^2 \right) p = Q_m \tag{5}$$

For the purposes of this simulation from the source plane wave was emitted and the equations describing the source of the acoustic wave took the form of:

$$-n \cdot \left( -\frac{1}{\rho_c} \left( \nabla_p - q_d \right) \right) + i \frac{k}{\rho_c} p + \frac{i}{2k\rho_c} \nabla_T p = Q_i \tag{6}$$

where $Q_i$ is given by:

$$Q_i = i\frac{k}{\rho_c}p_i + \frac{i}{2k\rho_c}\nabla_T p_i + n\frac{1}{\rho_c}\nabla p_i \tag{7}$$

and $p_i$ from equation (7) takes the form:

$$p_i = p_0 e^{-ik\frac{(r\cdot e_k)}{\|e_k\|}} \tag{8}$$

Using methods based on finite elements the entire space of the simulation must be divided into smaller fragments, for which calculations will be carried out. From the point of view of the simulation it is important, therefore, that these nodes are arranged in such a way as to take account of their increased density around the heterogeneity of the examined spatial area. Proper selection of the grid will help reduce the time needed to calculate acoustic wave behavior while maintaining high calculations precision.

## 3. Simulation setup

The simulation was performed using finite difference method in frequency domain. The simulation space was covered by a ultrafine mesh consisting of 29652 triangular elements. The simulations were performed for the ambient temperature of 293.15 K with the initial reference pressure of 1 atm. For band structure plots eigenfrequency studies were performed. Each of the cell boundaries satisfies the Bloch periodic conditions given by:

$$\psi(x + m\Lambda_x, y + n\Lambda_y, \omega) = \psi(x, y, \omega)e^{ikxm\Lambda_x}e^{ikxn\Lambda_y} \tag{9}$$

The two-dimensional phononic crystal structure used in this paper is presented in Fig. 1.

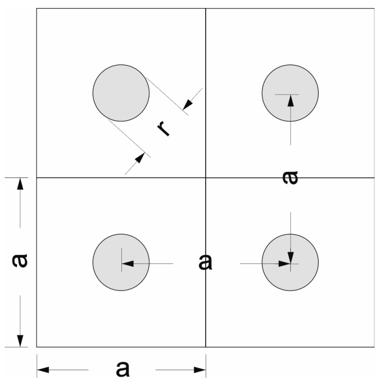


Fig. 1. 2D phononic crystal general dimensions

The $a$ value is the size of the unit cell in both directions (rectangular lattice) and the $r$ is the diameter of circular rod placed centrally inside the cell. According to this, the fill factor α will be defined as:

$$\alpha = \pi\left(\frac{r}{2}\right)^2 / a^2 \tag{10}$$

The first Brillouin zone with the $k$ wave vector path is presented in Fig. 2.

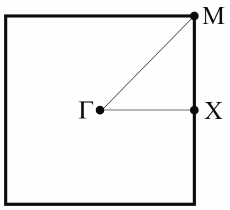


Fig. 2. First Brillouin zone with $k$ vector path

According to the presented Brillouin zone the path for $k$ vector was aligned along three main directions of Γ, X and M.

All result were presented for first nine bands or to the limit of 2250 Hz.

## 4. Results

Below in Figs. 3-9 the acoustic band structures for specific α parameter is presented.

As can be seen in Fig. 3 up to 2250 Hz the count of band curves for α=0.022 is eleven. In this region four band gaps were identified starting from 473.4 Hz up to 711.8 Hz for first band gap, from 1277.0 Hz to 1499.2 for second band gap, 1631 Hz to 2157 Hz third band gap, 1932.1 Hz to 2159.3 Hz the fourth band gap and last one located in this frequency range a narrow band ranging from 2159.3 Hz to 2176.0 Hz. In this and in all subsequent figures there is one straight line (the first one counting from bottom) at 1.28 Hz corresponding to the zeroth eigenfrequency. This low frequency range line is related to the overall cell dimensions and acoustic wave of this frequency can freely propagate through whole phononic crystals body almost without any attenuation. This is consistent with the current state of knowledge since it is known that very low frequencies can be transmitted over long distances. The general shape of acoustic band structure is well known for rectangular lattice and also in this case its appearance does not deviate from the expected one.

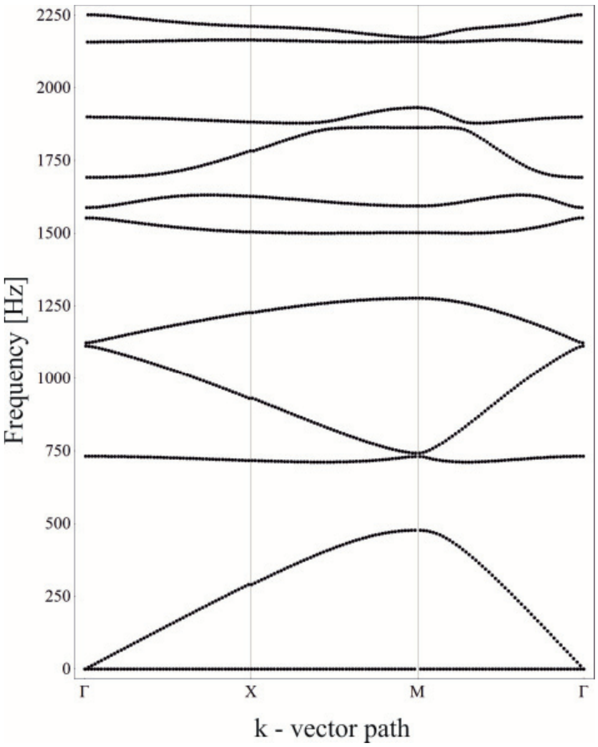


Fig. 3. Acoustic band structure for fill factor α=0.022

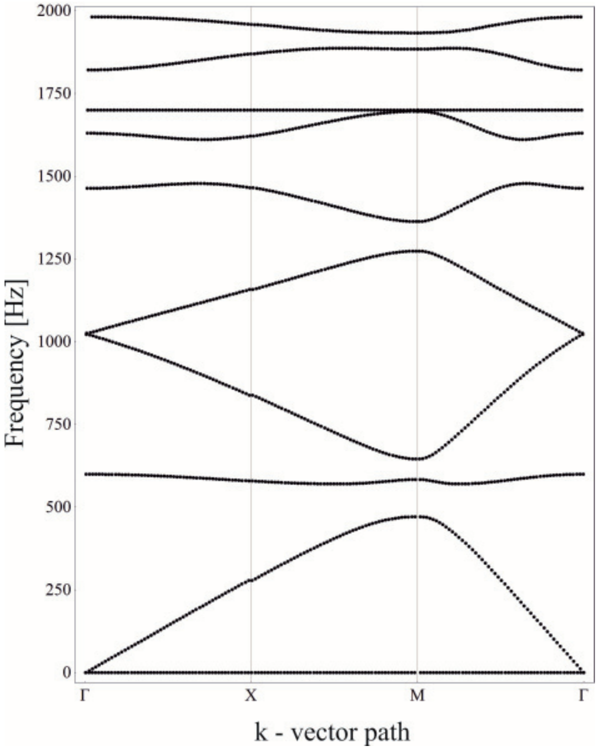


Fig. 5. Acoustic band structure for fill factor α=0.196

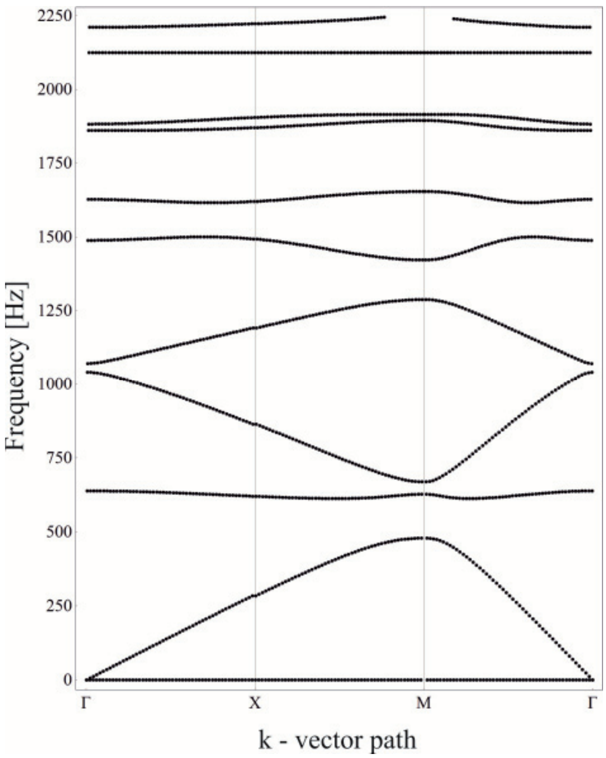


Fig. 4. Acoustic band structure for fill factor α=0.126

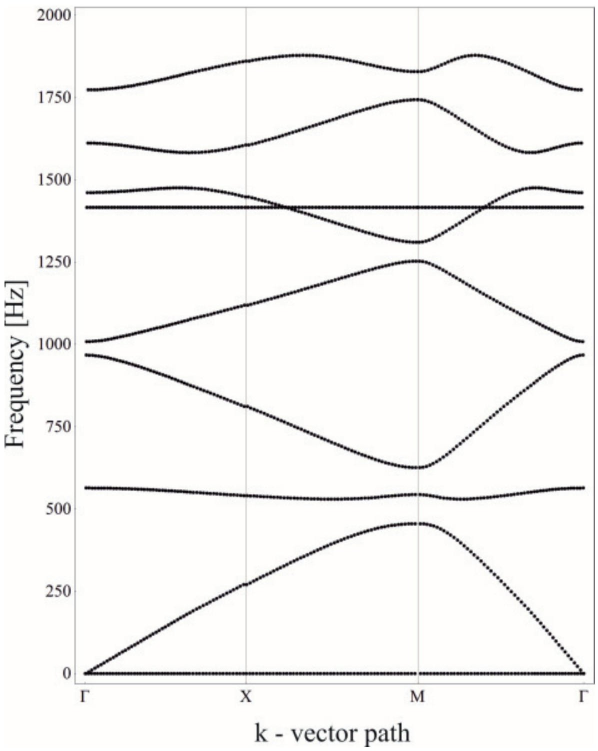


Fig. 6. Acoustic band structure for fill factor α=0.283

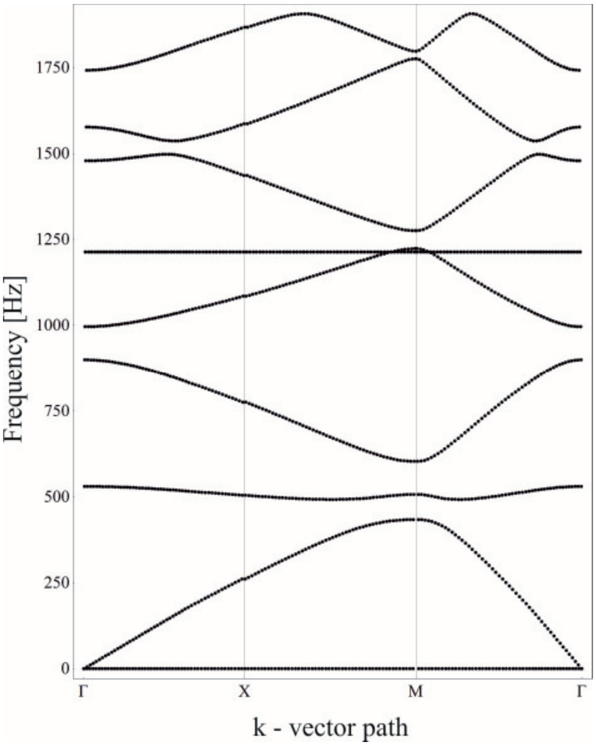


Fig. 7. Acoustic band structure for fill factor α=0.385

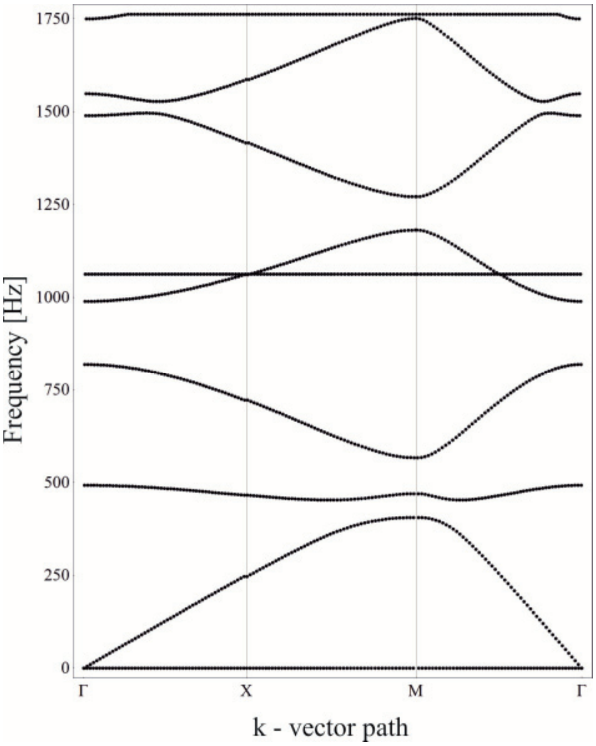


Fig. 8. Acoustic band structure for fill factor α=0.503

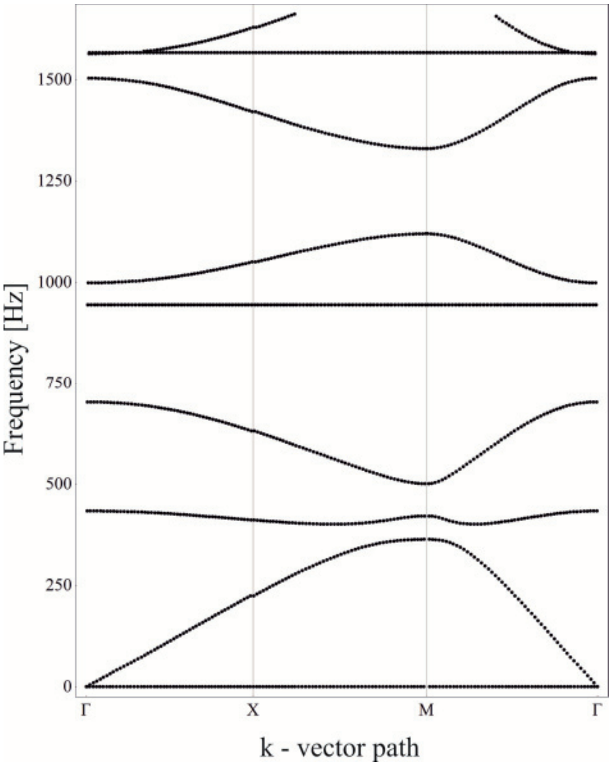


Fig. 9. Acoustic band structure for fill factor α=0.636

From the application point of view the most attractive are bands that have the lowest sensitivity to the direction change of incident acoustic wave. Ranges for these frequencies can be used in autocollimators (devices in which an acoustic wave propagates only in one specific direction regardless of incidence angle). In this case the most promising are those which lie near 750 Hz, 1550 Hz and about 2130 Hz.

The second important criterion for the application of the phononic crystal is the width of band gaps and their position. In the following figures can be seen the impact of changes in the diameter of the mercury rod on the size and position of the forbidden gaps and band line shapes.

Below in Table 1 data from analysis of Figs. 3-9 are collected for band gaps wider or equal to 29 Hz. Band gaps of smaller width are not considered to be significant for applications.

## 5. Discussion

As can be seen the influence of rods diameter is significant both to the bands width as well as for their positions and number. Most new forbidden band gaps appeared in the case of rod diameter $r$=040a (fill factor $\alpha$=0.126). A large number of band gaps is not always desirable, more often there is a need to have a widest possible bandwidth. For the studied case the widest (242 Hz) band gap was for rod diameter $r$=0.90a (or fill factor $\alpha$=0.636).

Table 1.
Data acquired from analysis of Figs. 3-9

| Rod diameter, a | Band gap number | Starting freq., Hz | End freq., Hz | Band width, Hz |
|---|---|---|---|---|
| 0.16 | 1 | 473 | 711 | 238 |
| | 2 | 1277 | 1499 | 222 |
| | 3 | 1631 | 1690 | 59 |
| | 4 | 1932 | 2159 | 227 |
| 0.40 | 1 | 478 | 610 | 132 |
| | 2 | 627 | 667 | 40 |
| | 3 | 1287 | 1421 | 134 |
| | 4 | 1500 | 1616 | 116 |
| | 5 | 1653 | 1860 | 207 |
| | 6 | 1915 | 2126 | 211 |
| | 7 | 2124 | 2210 | 86 |
| 0.50 | 1 | 471 | 569 | 98 |
| | 2 | 599 | 645 | 46 |
| | 3 | 1272 | 1365 | 93 |
| | 4 | 1479 | 1609 | 130 |
| | 5 | 1699 | 1821 | 122 |
| | 6 | 1888 | 1932 | 44 |
| 0.60 | 1 | 454 | 530 | 76 |
| | 2 | 541 | 626 | 85 |
| | 3 | 1251 | 1311 | 60 |
| | 4 | 1477 | 1583 | 106 |
| | 5 | 1743 | 1772 | 29 |
| 0.70 | 1 | 434 | 491 | 57 |
| | 2 | 507 | 605 | 98 |
| | 3 | 900 | 996 | 96 |
| | 4 | 1224 | 1275 | 51 |
| | 5 | 1498 | 1540 | 42 |
| 0.80 | 1 | 407 | 451 | 44 |
| | 2 | 470 | 566 | 96 |
| | 3 | 821 | 988 | 167 |
| | 4 | 1182 | 1271 | 89 |
| | 5 | 1498 | 1528 | 30 |
| 0.90 | 1 | 365 | 397 | 32 |
| | 2 | 422 | 499 | 77 |
| | 3 | 702 | 944 | 242 |
| | 4 | 944 | 997 | 53 |
| | 5 | 1119 | 1329 | 210 |
| | 6 | 1503 | 1566 | 63 |

The analysis of the positions of the bands showed that there are two effects associated with bands movement on the frequency axis: fission and compression effects. Near frequency 1000 Hz there is clearly visible fission especially evident when increasing rod diameter to $r$=0.6a (fill factor $\alpha$=0.283). First three bands (and clearly the zeroth band) are shifted toward lower frequencies while at the same time the rest (except one straight "traveling" band) is shifted toward higher frequencies. The second effect of compression is generally associated with fission effect and it consists of reducing the distance between the bands which is manifested also in reduction of the forbidden band gaps width. Also there are visible some deflections of bands that are considered to be nearly flat when the rod diameter is small (when $r$≤0.5a, two bands located near 1500 Hz).

One of the most promising band structures was observed for rod diameter r=0.40a where top six bands are nearly flat. Also worth noting are entirely straight bands visible on six of the seven acoustic band structure plots, due to the earlier mentioned application possibilities.

## 6. Conclusions

In this paper the influence of rod diameter on acoustic band gaps in 2D phononic crystal was investigated. As it was shown, the relationship between rod diameter and acoustic band structure is rather complicated. However, it can be certainly concluded that there are mainly two effects exerting the greatest influence on acoustic band structure behavior, that is: fission and band compression effects. Depending on the desired result, both of these effects may be desirable or not and the results shown in this study may help in the design of equipment using phononic crystals.

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